MA162: Finite mathematics

Jack Schmidt

University of Kentucky

April 17th, 2013

SCHEDULE:

- HW 1.1-1.4, 2.1-2.6, 3.1-3.3, 4.1, 5.1-5.3, 6A-6C, 7A (Late)
- HW 7B due Friday, Apr 19, 2013
- HW 7C due Friday, Apr 26, 2013
- Final Exam Tuesday, Apr 30, 2013 from 6pm to 8pm (new rooms on website)

Today we cover 7.5 ("what if")

7.5: The Punnet square of probability

- Suppose we have the following table of young men and women with and without driver's licenses:

	Yes	No	Total
M	491	9	500
F	486	14	500
T	977	23	1000

- What are the odds a randomly selected person has a driver's license?

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- What are the odds a randomly selected person is female?

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- Are females less likely to be drivers?

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- Are females less likely to be drivers?
- Probability a female is a driver: $\frac{486}{500} = 97\%$ nearly the same

7.5: Conditional probability

- Let's redo this using the language of events:
  - M is the event the chosen person is male
  - F is the event the chosen person is female
  - Y is the event the chosen person has a driver's license
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- We calculated it as $Pr(N \cap F)/Pr(N)$
- We need a name for this calculation, **conditional probability** $Pr(F|N) = Pr(N \cap F)/Pr(N)$ is the probability of F **given** N

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- We want to compare the probabilities of $Pr(A)$ versus $Pr(A|B)$ if they are equal then the events are **independent**

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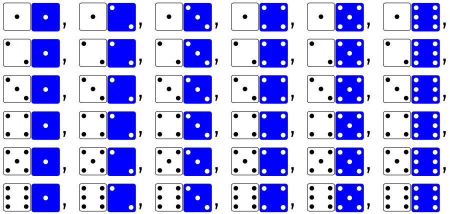
- Just count!



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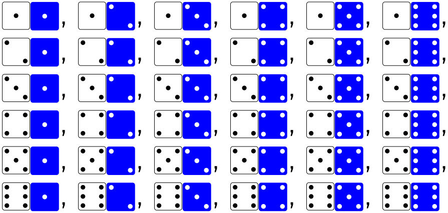

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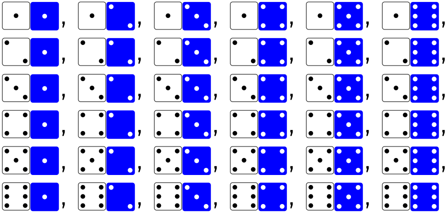

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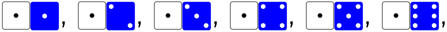

$4/6 \approx 67\%$

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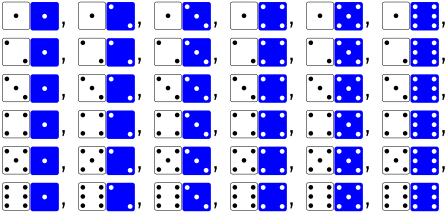

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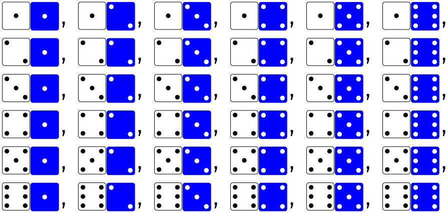

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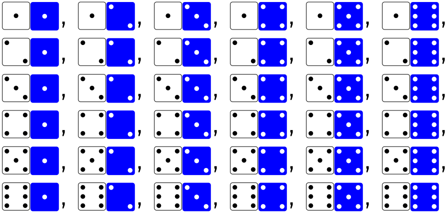

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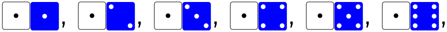

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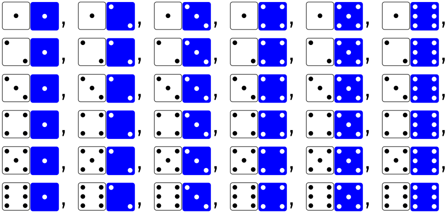

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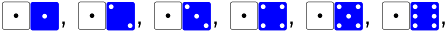

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- The first die had no effect on the outcome! The two events are said to be **independent**.

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  "Mostly". The probabilities are not equal, but they are close.

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  If it costs \$0.80 to play, how many chips would \$80.00 buy on average?

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- Weighted averages

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- What is the probability of getting a coke?
- 45%, right?

Reasoning backwards

- Shifty Teddy is spending some time on the gameshow “Who’s Gow?” and so you have to use his pal, Shifty Eddy, to run cokes for you. You end up with a coke 30% of the time. How often does he take the money and run?

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- $Pr(F) = 30\%$, and we want to find $Pr(E)$ which we calculated to be 60%, but where do we use the 50%?

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- The coke machine is 50% likely to give you a coke **IF** Eddy gives it the money, so we say $Pr(F|E) = 50\%$, the probability of F **given** E is 50%

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- The coke machine is 50% likely to give you a coke **IF** Eddy gives it the money, so we say $Pr(F|E) = 50\%$, the probability of F **given** E is 50%
- **Bayes's Law:** $Pr(E \cap F) = Pr(F|E) \cdot Pr(E)$ – a weighted average!

Practice exam

- A drug test is 98% accurate: out of 100 drug users, 98 will get a positive result, and 2 a negative; out of 100 non-users 98 will get a negative result, and 2 a positive. A company (somehow) knows that exactly 1 of its 100 employees is a drug user, but (somehow) does not know which one.
- An employee is picked at random to be tested, and tests positive. What is the probability that they are the drug user, given that they tested positive? Hint: It is NOT 98%.
- The company wants to be sure, and so tested the employee again. Positive. again. What is the probability that an employee is the drug user, given that they tested positive twice?

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- What is the probability that the drug test would correctly report on all 100 employees?
- An employee is picked at random to be tested twice, and tests positive once and negative once. What is the probability an employee is the drug user, given that they tested positive once and negative once?