Final Test Review **MA 432 S06**

The final will be just over chapters 7 and 8. So be able to do line and surface integrals of all types. Also know general facts about parametrizable surfaces such as how to find an element of surface area, and how to find the tangent plane to such a surface at a given point. Know the three main theorems in chapter 8 : Green's theorem, Stokes theorem, Divergence theorem, and be able to do integrals using these theorems. Finally be able to tell when a line integral is inpendent of path in an open set and how to find gradient fields. The following problems from the end of chapters 7 and 8 are good problems to practice on.
chapter 7 (page 514), 1, 3, 5, 7, 9, 11, 15, 17, 21, 23, 27.
chapter 8 (page 605) 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21.

Practice Test 1

25 pts 1. (a) Find $\int_C xy^2\,dx - x^2\,dy$, where C is the arc of the hyperbola, $y = \frac{1}{x}$, $1 \le x \le e$.

(b) Find a scalar function f so that $\nabla f = \mathbf{F}$, where
$\mathbf{F} = xe^{xy}(xy+2)\mathbf{i} + (x^3e^{xy} + 2ye^{y^2})\mathbf{j}$.

(c) Find $\int_C \mathbf{F}\cdot d\mathbf{s}$, where $\mathbf{F}$ is as in (b) and C is the curve, $x = t^5, y = t^4$, $0 \le t \le 1$.

(d) Find $\int_C (e^{x^2} + 3xy)\,dx + (\ln(y^2+4) + 3x)\,dy$ where C is the triangle with vertices : (2, 0), (0, 1), and (0, 0) oriented counterclockwise.

25 pts 2. Find $\int\int_S (y^2 + z)\,dS$, where S is that part of the sphere $x^2 + y^2 + z^2 = 8$ which lies inside the cone $z = \sqrt{x^2 + y^2}$.

25 pts 3. Find $\int\int_S \mathbf{F}\cdot\mathbf{n}\,dS$ where $S = \{(x, y, x^2 + y^2) : x^2 + y^2 \le 1\}$, $\mathbf{n}$ is the unit normal to S with positive $\mathbf{k}$ component and $\mathbf{F}(x, y, z) = (x, y, z)$.

25 pts 4. Find $\int\int_S (\nabla\times\mathbf{F})\cdot\mathbf{n}\,dS$ where $\mathbf{F} = (-x^3,\ y^3 + 3z^2\sin z,\ e^y\sin z + x^4)$ and S is that portion of the sphere $x^2 + y^2 + z^2 = 1$ with $z \ge 1/2$.

25 pts 5. Use the more general version of the divergence theorem to find
$\int\int_S \nabla\phi\cdot\mathbf{n}\,dS$ where S is any smooth closed surface containing the origin, $\mathbf{n}$ is the outer unit normal to S, and $\phi = (x^2 + y^2 + z^2)^{-1/2}$.

10 pts **Extra Credit:** Use the divergence theorem to show that if u is harmonic in an open set whose boundary is a smooth surface S, then $\int\int_S \nabla u \cdot \mathbf{n}\, dS = 0$, where $\mathbf{n}$ is the outer unit normal to S.

Answers Practice Test 1

1. (a) e, (b) $f = x^2 e^{xy} + e^{y^2}$ (c) 2 e - 1, (d) 1.
2. $\frac{\pi}{3}(16 - 7\sqrt{2})$.
3. $-\pi/2$.
4. 0.
5. -4π.

EC. Can do in class if asked.

Practice Test 2

25 pts 1. Given the surface $S = \{(u^2 + v, v, u^3) : 0 < u < 2,\ 1 < v < 3\}$.
(a) Find the tangent plane to this surface at $u = 1, v = 2$.
(b) Find the surface area of S.

25 pts 2. Which of the following line integrals $\int_{\mathbf{c}} \mathbf{u} \cdot \mathbf{ds}$ are independent of path? For those that are evaluate the line integral from $(1, 0, 0)$ to $(0, 0, 1)$.
(a) $\mathbf{u} = (e^{x^2}, x + 3(y + 3)^2(z - 1), (y + 3)^3)$.
(b) $\mathbf{u} = (xe^{x^2}, 3(y + 3)^2(z - 1), (y + 3)^3)$.
(c) $\mathbf{u} = (2xyz^3 + z, x^2z^3, 3x^2yz^2 + x)$.

25 pts 3. Use Green's theorem to find $\int_C \mathbf{F} \cdot \mathbf{ds}$ where
(a) C is the parabola $y = 1 - x^2$ and the line segment $y = 0, -1 \leq x \leq 1$ oriented counterclockwise and $\mathbf{F} = (x^2y, -xy^2)$.
(b) C is the rectangle with vertices $(\pm a, \pm b)$ oriented counterclockwise and $\mathbf{F} = (x^3, -xy^3)$.
(c) C is any smooth closed curve and $\mathbf{F} = (f(x), g(y))$ where f, g are continuously differentiable.

25 pts 4. Find $\int\int_S (\nabla \times \mathbf{v}) \cdot \mathbf{n}\, dS$, where $\mathbf{v} = (-y + x^2)\mathbf{i} + \mathbf{j} + xyz^2\mathbf{k}$ and S is that portion of the ellipsoid $x^2 + \frac{1}{4}y^2 + \frac{1}{9}z^2 = 1$, with $z \geq 0$, while $\mathbf{n}$ is the normal to S with positive $\mathbf{k}$ component.

25 pts 5. Find $\int\int_S \mathbf{F} \cdot \mathbf{n} dS$ where S is the surface of the ice-cream cone, $x^2 + y^2 + z^2 = 1, z \geq 0$, and $(z+1)^2 = x^2 + y^2, z \leq 0$, while $\mathbf{F} = xy\,\mathbf{i} + yz\,\mathbf{j} + xy\,\mathbf{k}$ and $\mathbf{n}$ is the outer unit normal to the solid bounded by this surface.

10 pts **Extra Credit:** Given the solid W bounded by the smooth surface S. Suppose that h is harmonic in W and has continuous second partials in the closure of W. If $h \equiv 0$ on S, use the divergence theorem to show that $h \equiv 0$ in W, provided W has the property that any two points in W can be joined by a curve, $\mathbf{c}$, consisting of a finite number of line segments which are parallel to the coordinate axes.
Hint: Use the vector field, $h\nabla h$ in the divergence theorem.

Answers Practice Test 2

1. (a) - 3 x + 3 y + 2 z + 1 = 0, (b) $\frac{8}{27}(19^{3/2} - 1)$.
2. (a) not independent of path since $\nabla \times \mathbf{u} \not\equiv 0$.
(b) independent of path, line integral equals, (55 - e)/2
(c) independent of path, line integral equals 0.
3. (a) - 4/7, (b) 0, (c) 0.
4. 2π.
5. $\pi/6$.
EC. Can do in class if asked.