Answers Test 2 F09 MA 213

1. (a) $\mathbf{v}(t) = \mathbf{r}\,'(t) = \int_0^t (4\tau\,\mathbf{i} + 2\,\mathbf{j} + 3\tau^2\,\mathbf{k}\,)\,d\tau + \mathbf{i} - 2\,\mathbf{k} = (2t^2+1)\,\mathbf{i} + 2t\,\mathbf{j} + (t^3-2)\,\mathbf{k}$.
So $\mathbf{r}'(1) = \mathbf{v}(1) = \langle 3, 2, -1\rangle$ and $v = |\,\mathbf{r}\,'(1)| = \sqrt{14}$.

(b) At $t = 1$, $\mathbf{r}\,' \times \mathbf{r}\,'' = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 2 & -1 \\ 4 & 2 & 3 \end{vmatrix} = \langle 8, -13, -2\rangle$.

Thus $\kappa = \frac{|\,\mathbf{r}\,'\times\,\mathbf{r}\,''|}{|\,\mathbf{r}\,'|^3} = \frac{\sqrt{237}}{14\sqrt{14}} \approx .293888$

2. (a) $\lim_{t\longrightarrow 0} g(0,t) = \lim_{t\longrightarrow 0} t^{-1/2} = \infty \neq g(0,0) = 0$, so g is not continuous at (0,0).
(b) $g_t = -(1/2)t^{-3/2}e^{-x^2/4t} + (1/4)x^2t^{-5/2}e^{-x^2/4t}$
(c) $g_x = -(x/2)t^{-3/2}e^{-x^2/4t}$.
(d) $g_{xx} = -(1/2)t^{-3/2}e^{-x^2/4t} + (1/4)x^2t^{-5/2}e^{-x^2/4t}$.
Therefore, $g_t = g_{xx}$ when $t > 0$.

3. $\nabla T = 32(10 - x^2 - y^2 - z^2)^{-2}\langle x, y, z\rangle$ so $\nabla T(1,1,2) = 2\langle 1,1,2\rangle$.
(a) $D_{\mathbf{u}}\,T(1,1,2) = \nabla T(1,1,2)\cdot\mathbf{u} = \frac{2}{3}(2-1+4) = 10/3$.
(b) In the direction of $\nabla T(1,1,2) = 2\langle 1,1,2\rangle$.
(c) Since $\nabla T(1,1,2)$ is a normal to the tangent plane through (1, 1, 2), the equation of the tangent plane is $2(x-1) + 2(y-1) + 4(z-2) = 0$ or $2x + 2y + 4z = 12$.

4. The critical points of f are when $f_x = 3x^2 - 6y = 0$, and $f_y = -6x + 24y^2 = 0$. Solving these equations: $2y = x^2$, $-6x + 6x^4 = 0$, so $x = 0, 1$. The critical points are $(0,0)$ and $(1, 1/2)$. Also $f_{xx} = 6x$, $f_{yy} = 48y$, $f_{xy} = -6$. Thus $D = D(x,y) = 288xy - 36$. Now $D(0,0) = -36 < 0$, so (0, 0) is a saddle point. Finally $D(1, 1/2) = 108 > 0$, and $f_{xx}(1, 1/2) = 6 > 0$ so (1, 1/2) is a local minimum point of f.

EC. Let $H(u,v,w) = h(u/w, v/w)$. By the chain rule, $H_u = w^{-1}H_x(u/w, v/w)$, $H_v = w^{-1}h_y(u/w, v/w)$, and $H_w = -uw^{-2}h_x(u/w, v/w) - vw^{-2}h_y(u/w, v/w)$. Thus $uH_u + vH_v + wH_w = uw^{-1}h_x + vw^{-1}h_y - uw^{-1}h_x - vw^{-}h_y = 0$ when $w \neq 0$.