Answers Test 3 **MA 213 F09**

1. (a) $D = \{(x, y) : 0 < y < x^2,\ 0 < x < 1\}$. Can sketch in recitation if asked.
(b) and (c) $I = \int_0^1 \int_0^{x^2} \frac{1}{1+x^3}\, dydx = \int_0^1 \frac{x^2}{1+x^3} dx =$ (let $u = 1+x^3$) $= \frac{1}{3}\ln(1+x^3)\,|_0^1 = \frac{1}{3}\ln 2 \approx$ .23105.

2. (a) $\int_D \int e^{x^2+y^2} dA = \int_0^{2\pi} \int_0^3 e^{r^2}\, rdrd\theta.$
(b) Above integral equals, $\int_0^{2\pi} (1/2)e^{r^2}|_0^3\, d\theta = \int_0^{2\pi} (1/2)[e^9 - 1]\, d\theta = \pi(e^9 - 1) \approx 25,453.45.$

3. (a) Can sketch in recitation if asked.
In (b) and (c) there are several possible iterations. Here is one such iteration for m and M_{xy}.
(b) $m = \int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{x^2+y^2}^{2-x^2-y^2} \sqrt{x^2+y^2}\, dzdydx$
(c) $M_{xy} = \int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{x^2+y^2}^{2-x^2-y^2} z\sqrt{x^2+y^2}\, dzdydx$
(d) $\bar{z} = \frac{M_{xy}}{m}$.

4. (a) and (b) $\int\int\int_E z\sqrt{x^2+y^2}\, dV = \int_0^{2\pi} \int_0^{\pi/2} \int_0^2 \rho^4 \sin^2\phi\, \cos\phi\, d\rho d\phi d\theta$
$= \int_0^{2\pi} \int_0^{\pi/2} (1/5)\rho^5\,|_0^2 \sin^2\phi\, \cos\phi d\phi d\theta = \frac{32}{5}\int_0^{2\pi} \frac{1}{3}\sin^3\phi|_0^{\pi/2}\, d\theta = \frac{32}{15}\,\theta|_0^{2\pi} = \frac{64\pi}{15} \approx 13.4.$

EC. Let $(0,0), (a,0), (0,b)$ be the endpoints of the legs of the right triangle. The line through $(a,0)$ and $(0,b)$ has the equation $\frac{x}{a} + \frac{y}{b} = 1$. Since (3,2) lies on this line we have $3/a + 2/b = 1$. The area of the triangel is $ab/2$. Using the method of Lagrange multipliers we look for critical points of $f(x,y) = \frac{1}{2}ab - \lambda(\frac{3}{a} + \frac{2}{b})$. Thus, $f_a = \frac{b}{2} + \frac{3\lambda}{a^2} = 0$ and $f_b = \frac{a}{2} + \frac{2\lambda}{b^2} = 0$. Solving for λ gives $\frac{a^2b}{6} = -\lambda = \frac{ab^2}{4}$ so $a = \frac{3}{2}b$. Using the constraint yields $4/b = 1$ so $b = 4$ and $a = 6$ are the lengths of the legs of the area minimizing right triangle.