Test Review 2 **MA 213** **Fall 09**

The test will be over chapter 13.3, 13.4, and 14.1 - 14.7. I suggest you begin your review by going over your homework, quizzes and notes. In chapter 13, given a parametrication or position vector $\mathbf{r(t)}, a \le t \le b$, of a curve C know what is meant by and be able to find: the arclength function, the length of a curve, curvature, the tangential and normal components of acceleration, and the unit vectors $\mathbf{T}, \mathbf{N}$. Be able to work problems on velocity and acceleration similar to your homework. Good review problems from Stewart for 13.3, 13.4 begin on page 849: Concept Check, 5, 6, 7, 8, True - False Quiz 4, 5, 6,7,8,9, Exercises, 11, 13, 17, 19.

In chapter 14 know what is meant by such symbols and phrases as : the domain of $f(x, y)$; $\lim_{(x,y)\longrightarrow(a,b)} f(x, y) = L$; f is continuous at (a, b); f_x, f_y, f_{xy}. Be able to find limits and take partial derivatives of functions. You should know what is meant by the tangent plane approximation of a function of several variables (relative to a given point). Learn the difference between: continuity, partials existing, and differentiability of a function of several variables (again relative to a given point). Know the chain rule in its various forms and also some applications of the chain rule. Learn the definition of the gradient of a function and its use in finding directional derivatives, especially directions of most rapid increase/decrease, as well as tangent planes to surfaces. Be able to find the critical points of a function of two variables and memorize the second derivative test for determining relative extrema. Also you should know how to solve word problems involving absolute maxima or minima. Good review problems for chapter 14 begin on page 944: Concept Check, 3, 5, 7, 9, 11, 15, 17. True - False Quiz, 1, 3, 5, 7, 9, 11. Exercises : 1, 3, 9, 13, 15, 17, 19. 21, 23, 25, 27, 29, 31, 33, 35, 39, 43, 45, 47, 51, 53, 55, 63.

Homework Quiz problems for the next testing period from which homework quiz questions will usually be chosen follow.

Homework

Section 14.8 (page 940) 5, 9, 11, 19, 27, 31, 35
Section 15.1 (page 958) 3, 11
Section 15.2 (page 964) 3, 9, 15, 21, 27, 35
Section 15.3 (page 972) 5, 11, 15, 21, 27, 31, 39, 45
Section 15.4 (page 978) 7, 11, 15, 17, 21, 25, 29, 35
Section 15.5 (page 988) 3 ,7, 11, 17
Section 15.6 (page 998) 5, 11, 17, 21, 37, 43.
Section 15.7 (page 1004) 1, 3, 7, 11, 15, 21, 23, 27
Section 15.8 (page 1010) 1, 3, 5, 7, 13, 17, 21, 23, 29, 33 (a). 39.
Section 15.9 (page 1020) 1, 7, 11, 13, 17 (a), 21.
Section 16.1 (page 1032) 5, 21, 29.
Section 16.2 (page 1042) 1, 3, 7, 11, 19, 33, 41.
Section 16.3 (page 1053) 5, 9, 13, 17, 21. Section 16.4 (page 1060) 5, 7, 11, 13, 17.

Here are two practice tests (complete with answers), quite similar to the actual test.

Practice Test 1

25 pts 1. The position vector for a curve at time t is given by
$\mathbf{r}\,(t) \;=\; \frac{t^2}{2}\;\mathbf{i}\;+\;\frac{t^3}{3}\;\mathbf{j}\;+\;\frac{t^4}{4}\;\mathbf{k}$. Find
(a) A unit vector, $\mathbf{T}$, tangent to the curve at the point corresponding to $t = 1$.
(b) The curvature κ of the curve at the point corresponding to $t = 1$.

25 pts 2. Let $k(x, y) = \ln(x^2 + y^2)$. Show that k is a solution to Laplace's equation whenever $(x, y) \neq 0$. That is show for $(x, y) \neq (0, 0)$, $\Delta k = \dfrac{\partial^2 k}{\partial y^2} + \dfrac{\partial^2 k}{\partial x^2} = 0$.

25 pts 3. The potential, P of a unit charge at the origin is given by : $P(x, y) = \sqrt{x^2 + y^2}$, for $(x, y) \neq (0, 0)$. Use the tangent plane approximation to P to find the approximate value of P at the point (3.1, 4.2).

25 pts 4. Use the chain rule to find $\frac{dz}{dt}$ at $t = \frac{\pi}{2}$ when $z = x\cos xy$, $x = t^2$, $y = \dfrac{1}{t}$.

10 pts **Extra Credit:** Find the absolute maximum of $h(x, y) = x^2 + xy^2 + 2y^2$ subject to the constraint $x^2 + y^2 = 1$.

Answers Practice Test 1

1. (a) At $t = 1$, $\mathbf{T} = (1/\sqrt{3})\,\langle 1, 1, 1\rangle$.
(b) $\kappa = \frac{|\,\mathbf{r}\,' \times \mathbf{r}\,''|}{|\,\mathbf{r}\,'|^3} = \sqrt{2}/3$.

2. $k_{xx} + k_{yy} = \frac{2(y^2-x^2)}{(x^2+y^2)^2} + \frac{2(x^2-y^2)}{(x^2+y^2)^2} = 0$ when $(x, y) \neq (0, 0)$.

3. $P(3.1, 4.2) \approx 5.22$.

4. $dz/dt = -\pi^2/4$.

EC. Absolute maximum = 59/27

Practice Test 2

25 pts 1. The position vector for a curve at time t is given by
$\mathbf{r}\,(t) \;=\; \frac{t^2}{2}\;\mathbf{i}\;+\;\frac{t^3}{3}\;\mathbf{j}\;+\;\frac{t^4}{4}\;\mathbf{k}$. Find the tangential and normal components of acceleration of a particle moving along this curve.

25 pts 2. Find the following:
(a) The domain of g when $g(x, y) = \frac{\sqrt{4-x^2-y^2}}{x+y}$. Also sketch this domain.
(b) $\lim\limits_{(x,y)\to(0,0)} \frac{x^3}{x^2+y^2}$ if it exists. Justify your reasoning.
(c) $\lim\limits_{h\to 0} \frac{k(3+h,\pi/4)-k(3,\pi/4)}{h}$ if it exists, when $k(x, y) = 3x^2\cos y$.

25 pts 3. Given the function, f, defined by: $f(x, y) = x\,e^{2y} + 2\,x^2 + 3\,y$. Find:
(a) The directional derivative of f at (3, 0) in the direction of the vector, $\mathbf{v} = \langle -1, 4\rangle$.
(b) The direction in which f is decreasing most rapidly at (3, 0) .
(c) The most rapid rate of decrease of f at (3, 0).
(d) The tangent plane to the surface, $z = f(x, y)$, at (3, 0, 21)

(e) The linear or tangent plane approximation to f at (3,0,21).

25 pts 4. Given the function, $f(x,y) = \frac{2}{3}x^3 - 4xy + 2y^2 - 6y - 4$.
(a) Find the critical points of f.
(b) Use the second derivative test for functions of several variables to determine whether f has a local minimum, maximum , or neither at each critical point.

10 pts **Extra Credit** Suppose that f, g are differentiable for all real numbers and $w = f(u) + g(v)$ where $u = x + y$, $v = x - y$. Show that $w_x\, w_y = (w_u)^2 - (w_v)^2$.

Answers Practice Test 2

1. $a_T = \frac{\mathbf{a} \cdot \mathbf{v}}{v} = \frac{6}{\sqrt{3}}$, and $a_N = \frac{|\,\mathbf{v} \times \mathbf{a}\,|}{v} = \sqrt{6}/\sqrt{3}$.

2. (a) Domain of $g = \{(x,y) : x \neq -y \text{ and } x^2 + y^2 \leq 4\}$. Can graph in class if asked.
(b) 0 as one sees from using the squeeze theorem and comparing with $\pm|x|$.
(c) $k_x(3, \pi/4) = 18/\sqrt{2}$

3. (a) $\mathbf{u} = \langle -1, 4\rangle/\sqrt{17}$, so $D_{\mathbf{u}}f = 23/\sqrt{17}$
(b) f is decreasing most rapidly in the direction of $-\nabla f(3,0) = -\langle 13, 9\rangle$
(c) maximum rate of decrease = - $\sqrt{250}$
(d) $13x + 9y - z - 18 = 0$.
(e) $L(x,y) = 13x + 9y - 18$ is the linear approximation to f.

4. Critical points are (3, 9/2), (- 1, 1/2). (3, 9/2) is a relative minimum and (- 1, 1/2) is a saddle point by the second derivative test for functions of several variables.

EC. By the chain rule, $w_x = w_u + w_v$, $w_y = w_u - w_v$, so $w_x\, w_y = w_u^2 - w_v^2$.