The test is over section 14.8 and 15.1 - 15.8. I suggest you begin your review by going over your homework, quizzes and notes. In chapter 14 be able to solve max/min problems using Lagrange multipliers. Review problems in Stewart, page 947, on Lagrange multipliers are 59, 61, 63.
To give a brief outline of chapter 15, know the definition of the double integral and some of its properties. Know what is meant by and how to evaluate iterated integrals. Be able to find double integrals over general regions by expressing the integral as an integral (or sum of integrals) over type 1 or type 2 regions. Also know how to iterate and find integrals over polar regions. Know the definition of the triple integral. Practice iterating and evaluating triple integrals. Know some of the applications of double and triple integrals to areas, volumes, moments, center of mass, moment of inertia. Be able to find integrals in cylindrical and spherical coordinates. I will give you the expressions for x, y, z and an element of volume in each coordinate system, if needed on a given problem. Problems from the review at the end of chapter 15 (page 1021) are Concept Check: 3,7. True - False Quiz: 1,3,5,7. Exercises: 5, 7, 9, 11, 13, 17, 19, 23, 25, 29, 35, 39, 40. Homework for the rest of the semester follows.

Homework

Section 15.9 (page 1020) 1, 7, 11, 13, 17 (a), 21.
Section 16.1 (page 1032) 5, 21, 29.
Section 16.2 (page 1042) 1, 3, 7, 11, 19, 33, 41.
Section 16.3 (page 1053) 5, 9, 13, 17, 21.
Section 16.4 (page 1060) 5, 7, 11, 13, 17.
Section 16.5 (page 1060) 3, 7, 13, 17, 31.

Here are two practice tests (complete with answers), quite similar to the actual test.

Practice Test 1

25 pts 1. Given the region, D, bounded by the parabolic arc, $y = x^2, x > 0$, the line $y = -x + 2$, and the y axis. If $f(x, y) = 3 + 2y$
(a) Write $\int\int_D f(x, y)\, dA$ as an iterated integral over a type one region.
(b) Write $\int\int_D f(x, y)\, dA$ as a sum of integrals over type two regions.
(c) Evaluate the double integral you wrote in (a).

25 pts 2. Given polar coordinates $x = r\cos\theta, y = r\sin\theta$, and $dA = rdrd\theta$. Let D be the domain inside the loop of the rose curve $r = \cos 2\theta$ which is symmetric with respect to the positive x axis. Find $\int\int_D \sqrt{x^2 + y^2}\, dA$.

25 pts 3. Given the solid E bounded by the planes, $x = 0, y = 0, z = 0$ and $x + y + z = 2$. The mass density of this solid is equal to 1.
(a) Set up the integrals for the coordinate $\bar{x}$ of the center of mass of E.
(b) If the mass or volume of E is $4/3$, find $\bar{x}$.

25 pts 4. Find the volume of the solid bounded below by the cone $z = 3\sqrt{x^2 + y^2}$ and above by the paraboloid $z = 4 - x^2 - y^2$. Hint: use cylindrical coordinates, $x = r\cos\theta, y = r\sin\theta, z = z, dV = rdzdrd\theta$.

10 pts **Extra Credit.** Use Lagrange multipliers to find the absolute maximum of $h(x,y) = x^2 + xy^2 + 2y^2$ subject to the constraint $x^2 + y^2 = 1$.

Answers Practice Test 1

1. (a) $\int_0^1 \int_{x^2}^{2-x} (3+2y)dydx$

(b) $\int_0^1 \int_0^{\sqrt{y}} (3+2y)dxdy + \int_1^2 \int_0^{2-y} (3+2y)dxdy$

(c) 169/30.

2. 2/9.

3. (a) $M_{yz} = \int_0^2 \int_0^{2-x} \int_0^{2-x-y} xdzdydx$, and $m = \int_0^2 \int_0^{2-x} \int_0^{2-x-y} dzdydx$ so $\bar{x} = M_{yz}/m$.

(b) $M_{yz} = 2/3$ so $\bar{x} = 1/2$.

4. Volume $= \int_0^{2\pi} \int_0^1 \int_{3r}^{4-r^2} dzrdrd\theta = 3\pi/2$.

EC. Absolute maximum = 59/27

Practice Test 2

25 pts 1. Use Lagrange multipliers to find the points on the surface $y^2 - xz = 9$ that are closest to the origin.

25 pts 2. Given the iterated integral, (*) $\int_0^2 \int_1^{e^x} 16\, y^3\, dydx$.

(a) Graph the region D over which the double integral is taken.

(b) Write an equivalent double integral in which the order of integration is reversed.

(c) Evaluate the integral in (*). Leave your answer in terms of exponentials.

25 pts 3. Given the double integral, $\int\int_D \frac{y+1}{\sqrt{x^2+y^2}}\, dxdy$ where D is the region inside the circle $(x-1)^2 + y^2 = 1$. (a) Write an equivalent iterated double integral in polar coordinates.

(b) Evaluate the integral in (a) .

25 pts 4. Given $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} (x^2+y^2+z^2)^{1/2}\, dV$ and spherical coordinates, $x = \rho\cos\theta\sin\phi, y = \rho\sin\theta\sin\phi, z = \rho\cos\phi, dV = \rho^2\, \sin\phi d\rho d\phi d\theta$.

(a) What solid E is this integral being integrated over ?

(b) Write an equivalent triple integral in spherical coordinates.

(c) Evaluate the integral in (b).

10 pts **Extra Credit.** Find the mass of the solid E in 4 (a) if the mass density of this solid is $\rho(x,y,z) = z$.

Answers Practice Test 2

1. $(0, \pm 3, 0)$

2. (a) $D =$ the region bounded by the curves, $y = e^x, y = 1, x = 0, x = 2$. Can sketch in class.

(b) $\int_1^{e^2} \int_{\ln y}^{2} 16\, y^3\, dx dy.$

(c) $e^8 - 9.$

3. (a) $\int_{-\pi/2}^{\pi/2} \int_0^{2\cos\theta} (r \sin\theta + 1)\, dr\, d\theta.$

(b) 4.

4. (a) $E = \{(x, y, z) : x^2 + y^2 + z^2 < 1 \text{ and } x, y, z > 0\} =$ the part of the unit ball in the first octant.

(b) and (c) $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} (x^2 + y^2 + z^2)^{1/2}\, dV \quad = \quad \int_0^{\pi/2} \int_0^{\pi/2} \int_0^1 \rho^3 \sin\phi\, d\rho d\phi d\theta = \pi/8.$

EC. $m = \pi/16.$