Answers Test 3 **MA 214 F 10**

1. Using variation of parameters a particular solution Y to $LY = t$ can be found of the form $Y = u_1 + tu_2 + t^{1/2}u_3$ where u_1, u_2, u_3 are functions of t whose derivatives satisfy the equations, (a) $u_1' + tu_2' + t^{1/2}u_3' = 0$, (b) $u_2' + t^{-1/2}u_3'/2 = 0$, (c) $-t^{-3/2}u_3'/4 = t$. From (c) it follows that $u_3' = -4t^{5/2}$, so integrating one gets $u_3 = -(8/7)t^{7/2}$. Using (b) to solve for u_2' in terms of u_3' and then substituting for u_3', one obtains $u_2' = 2t^2$. Integrate to find $u_2 = 2t^3/3$. Finally using (a) to solve for u_1' in terms of u_2', u_3' and then substituting for u_2', u_3' one deduces $u_1' = 2t^3$. Integrate to find that $u_1 = t^4/2$. Thus $Y = t^4/2 + 2t^4/3 - 8t^4/7 = t^4/42$.

2. (a) $\mathcal{L}^{-1}\left[se^{-s}/(s^2+16)\right] = u_1(t)\cos(4t-4)$.
(b) $\mathcal{L}^{-1}\left(\frac{1}{s^2+4s+8}\right) = \mathcal{L}^{-1}\left(\frac{1}{(s+2)^2+4}\right) = (1/2)\,e^{-2t}\sin(2t)$.
(c) $\mathcal{L}f = e^{-12}\,\mathcal{L}\left(u_3\,e^{-4(t-3)}\right) = e^{(-3s-12)}(s+4)^{-1}$.

3. (a) $\mathcal{L}(y' + 2y) = (s+2)\mathcal{L}y - 5 = 3(s+1)^{-1}$ so , $\mathcal{L}\,y = \frac{3}{(s+2)(s+1)} + \frac{5}{s+2} =$ (by partial fractions) $\frac{3}{s+1} + \frac{-3}{s+2} + \frac{5}{s+2}$. Taking inverse Laplace Transforms : $y = 3e^{-t} + 2e^{-2t}$.

(b) $\mathcal{L}(y'' + 6y' + 9y) = (s^2+6s+9)\mathcal{L}\,y - 1 = 9/s$. Thus $\mathcal{L}\,y = \frac{s+9}{s(s+3)^2} = \frac{A}{s} + \frac{B}{(s+3)^2} + \frac{C}{(s+3)} = \frac{A(s+3)^2 + Bs + C(s+3)s}{s(s+3)^2}$. Equate numerators to obtain, $s+9 = A(s+3)^2 + Bs + C(s^2+3s)$. Evaluate at $s=-3$: get $-3B = 6$, $B = -2$. Evaluate at $s = 0$: get $9A = 9$, $A = 1$. Comparing the coefficient of s^2 , it follows that $C = -A = -1$. Hence, $y = \mathcal{L}^{-1}\left(\frac{s+9}{s(s+3)^2}\right) = 1 - 2t\,e^{-3t} - e^{-3t}$.

EC. (a) Characteristic equation for $Ly = 0$ is $(r^2+2r+2)^2(r-2)^3 = 0$ So $r = 2$, three times and by the quadratic formula, $r = -1 \pm i$, twice. General solution to $Ly = 0$ is $y = (c_1 + c_2 t)e^{-t}\cos t + (c_3 + c_4 t)e^{-t}\sin t + (c_5 + c_6 t + c_7 t^2)e^{2t}$.

(b) Since 2 occurs 3 times as a root in the characteristic equation for $Ly = 0$ and t^2 is a polynomial of degree 2 it follows from the table in the book that Y has the form $Y = t^3(At^2 + Bt + C)e^{2t} = (At^5 + Bt^4 + Ct^3)e^{2t}$.

Homework Problems

Section 6.5 (page 345)	5, 7, 9
Section 6.6 (page 351)	5, 7, 9, 13, 25 (a), 27 (a)
Handout (page 366)	1, 5
Handout (page 369)	1, 5.