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Dear Colleague

I am writing this letter on behalf of Serban Costea who I understand is applying for an assistant professor and/or a post doc position in your department. Serban is a former PhD student of Professor Juha Heinonen at the University of Michigan, who tragically passed away last fall. I first met Professor Costea at the Albert Baernstein 65 th birthday conference at Washington University in St Louis. At that time Serban was still working on his thesis. He impressed me with his knowledge and enthusiasm for analysis in general. I am familiar with the following of his papers:

[1] *Strong A_∞ weights and scaling invariant Besov capacities,* Rev. Mat. Iberoamericana **23** (2007), no. 3, 1067-1114.

[2] *Scaling invariant Sobolev-Lorentz capacity on $\boldsymbol{R}^n$,* Indiana Univ. Math. Journal, **56** (2007), no. 6, 2641-2670.

[3] *Conductor inequalities and criteria for Sobolev-Lorentz two - weight inequalities,* with Vladimir Maz'ya, Sobolev Spaces in Mathematics II. Applications in analysis and partial differential equations, 103-122, Springer (2008).

[4] *Sobolev capacity and Hausdorff measures in metric measure spaces,* to appear Ann. Acad. Sci. Fenn.

[5] *Strong A_∞ weights, Besov, and Sobolev capacities in metric measure spaces,* to appear in Houston J. Math.

[6] *The Corona Theorem for the Drury - Arveson Hardy space and other holomorphic Besov-Sobolev spaces on the unit ball in $\mathbf{C}^\mathbf{n}$, submitted*

[7] *BMO Estimates for the H^∞ Corona Problem,* submitted

[1] is based on his thesis. In this paper Serban studies strong A_∞ weights and Besov capacities. One of his main results is to show that if the distributional gradient of a function, u, has small enough norm in a certain Lorentz space, then e^{nu} is a strong A^∞ weight with data depending only on n and p. This result generalizes work of his advisor and coauthors who proved similar results for spaces strictly contained in the above Lorentz space. In order to prove his results, Serban developed a complete theory of Besov capacities: His theory included a workable definition and basic properties

of such capacities - monotonicity, finite subadditivity, convergence and truncation results. He also studied the Hausdorff dimension of null sets and quasi- continuity of potentials, relative to Besov capacities.

In [2] he carried out the above program for a Sobolev type capacity defined using the Lorentz norm.

[4], [5], are generalizations of [1], [2], to certain metric measure spaces. The metric spaces are only assumed to be Ahlfors Q regular, geodesic, and to satisfy a $(1, s)$ Poincaré inequality for some $s \in (1, Q]$. To obtain analogues of the results in [1],[2], Serban had to master techniques developed for metric spaces by his advisor, J. Björn, J. Cheeger, J. Kinnunen, P. Koskela, O. Martio, and N. Shanmugalingam.

In [3] a capacity strong type inequality is proved for the capacities considered in [1], [2] (the terminology is due to D.R. Adams).

In [6], Serban and coauthors Sawyer, Wick, solve a long standing problem involving the Corona theorem in certain Besov - Sobolev spaces of the unit ball in $\mathbf{C^n}$. Serban's contribution was apparently his excellent knowledge of Besov-Sobolev spaces. In [7] new results are also obtained for the $H^\infty(B_n)$ Corona problem.

It is clear from his papers that Professor Costea is already in command of an impressive armada of harmonic analysis type techniques. In the future I expect that he will continue to widen his research areas and also continue to develop his already extensive knowledge of capacities, quasi-regular mappings, and non-linear PDE in metric spaces and beyond. Finally on a personal note, he has an enthusiasm for mathematical research that others can feed off of. Thus if you have a position for a budding young analyst, I hope you give Serban strong considereation.

Sincerely,

John Lewis