MA 162, Sections 1, 2, 3, 4
Example Problems for Exam 1

1. Solutions to Matrices in Parametric Form

Suppose the matrix below is the row reduced form of some matrix A,

$$\begin{pmatrix} 1 & 0 & 3 & 4 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Then the parametric form of the solution is obtained as follows:

(a) From the matrix, we can write equations obtained from the entries in each row.

$$\begin{aligned} x_1 + 0x_2 + 3x_3 &= 4 \\ 0x_1 + 1x_2 + 2x_3 &= 1 \\ 0x_1 + 0x_2 + 0x_3 &= 0 \end{aligned}$$

(b) Solving for x_1, x_2, and x_3 respectively gives,

$$\begin{aligned} x_1 &= -3x_3 + 4 \\ x_2 &= -2x_3 + 1 \\ x_3 & \text{ is free.} \end{aligned}$$

(c) Since x_3 is free, we let $x_3 = c$, where c is a real number. So the equations become,

$$\begin{aligned} x_1 &= -3c + 4 \\ x_2 &= -2c + 1 \\ x_3 &= c \end{aligned}$$

This is the parametric form of the solution. Remember that since we obtained $0 = 0$, this implies that the system has an infinite number of solution. Since x_3 was free, it can take on the value of any real number; also note that the values for the other variables depend on x_3. But after a value for x_3 is fixed, the values for the other two variables become fixed also.

2. Checking to see if a solution given in parametric form is a solution to a given system

Decide whether or not $[-1, (3 - 3t), t]$ is a solution to the follow system of equations.

$$\begin{aligned} 5x + 2y + 6z &= 1 \\ -2x + y + 3z &= 5 \end{aligned}$$

To see is the given parametric form is a solution, we need to obtain numbers from the parametric form. For convenience, choose $t = 1$. The a solution is:

$x = -1$

$y = 3 - 3(1) = 0$

$z = 1$

Substituting these values gives,

$5(-1) + 2(0) + 6(1) = 1$

$-2(-1) + (0) + 3(1) = 5$

Since the values for x, y, and z satisfy the equations, the parametric form given is a solution.

3. Given the following system, determine a value for k so that the system either has no solution or has infinite solutions.

$$3x - 2y = 5$$
$$kx + 4y = 7$$

In class, we solved problems similar to this by finding the slopes of the lines. Dr Sathaye has provided an example of doing this problem by using Cramer's Rule, which is the most simple way of solving the problem.

We can also solve the problem by using a matrix:

(a) First, write the system in matrix form.

$$\begin{pmatrix} 3 & -2 & 5 \\ k & 4 & 7 \end{pmatrix}$$

(b) Our goal is to have an equation of the form "$(k + B)x = A$" where A and B are some numbers. To do this, we should swap the rows of the matrix to obtain,

$$\begin{pmatrix} k & 4 & 7 \\ 3 & -2 & 5 \end{pmatrix}$$

(c) We can now use the -2 in the second row to "kill" the 4 in the first row. So $R1 : R1 + 2R2$,

$$\begin{pmatrix} k+6 & 0 & 17 \\ 3 & -2 & 5 \end{pmatrix}$$

(d) Now we can write an equation from the first row,

$$(k + 6)x = 17$$

(e) Looking at this equation we have to determine if we can get something of the form $0 = 0$", "$17 = 17$", etc, or if can get something of the form "$0 = 1$", "$0 = 17$", etc.

Since we still have the variable "x", we cannot choose a value for k that would guarantee a statement like $17 = 17$ in every case. On the other hand, we can choose a value for k that would give a contradiction in every case.

Specifically, we can choose $k = -6$. This gives,

$$(k+6)x = 17, \text{ substituting in } k = -6,$$
$$[(-6)+6]x = 17$$
$$0x = 17$$
$$0 = 17$$

So we have arrived at a contradiction. This says that if $k = 17$, then the system is inconsistent.