Homework

October 12, 2011

Let $\mathcal{C}(X,Y)$ be the set of continuous functions from a topological space X to a topological space Y. The following two observations will be used below.

1. There is a (set) function
$$\epsilon\colon X \times \mathcal{C}(X,Y) \to Y$$
defined by $(x,\phi) \mapsto \phi(x)$. This is called the *evaluation*.

2. Given a function $f\colon X \times Z \to Y$ there is a function
$$A(f)\colon Z \to \mathcal{C}(X,Y)$$
defined by $A(f)(z)(x) = f(x,z)$.

   Given a function $g\colon Z \to \mathcal{C}(X,Y)$ there is a function
$$B(g)\colon X \times Z \to Y$$
defined by $B(g)(x,z) = g(z)(x)$.

The *compact-open topology* on $\mathcal{C}(X,Y)$ has subbasis given by the sets
$$S(C,U) = \{f \in \mathcal{C}(X,Y) | f(C) \subset U\}$$
for all compact subsets C of X and open sets U of Y. When no topology is specified for $\mathcal{C}(X,Y)$ use the compact-open topology.

Prove the following theorems.

1. If X is a compact Hausdorff space ϵ is continuous.

2. If f (as in 2 above) is continuous then $A(f)$ is continuous. If X is a compact Hausdorff space and g (as in 2 above) is continuous then $B(g)$ is continuous.