

Homework

October 14, 2011

1. (a) What subsets of \mathbb{R} with the finite complement topology are compact?
(b) Let \mathbb{R} have the topology where the open sets are the empty set, \mathbb{R} , and sets whose complement is countable. Is $[0, 1]$ compact?
2. Show that if Y is compact the projection map $\pi: X \times Y \rightarrow X$ is closed. (Use the Tube lemma.)
3. Let $f: X \rightarrow Y$ be continuous and Y be compact Hausdorff. Then f is continuous if and only if the graph of f

$$\Gamma_f = \{(x, f(x)) | x \in X\}$$

is closed in $X \times Y$. (Use 2.)