HOMOTOPICAL ALGEBRA
MA 752 : TOPICS IN TOPOLOGY

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Two topological spaces are "the same" if they are homeomorphic. Two groups are "the same" if they are isomorphic. We know from practice that it is actually much more useful to think about homeomorphisms and isomorphisms than to require groups or topological spaces to be identical.

In many situations we want to be even more flexible about what objects we consider to be the same. For example, we many not want to distinguish between homotopy equivalent topological spaces or even spaces that are weakly homotopy equivalent. In algebra we may want to think of chain homotopy equivalent chain complexes or quasi isomorphic chain complexes as the same.

Homotopy equivalences and chain homotopy equivalences have inverses only up to homotopy. Quasi isomorphisms and weak homotopy equivalences have inverses only after taking homology or looking at homotopy groups. None of these are honestly invertible like isomorphisms and homeomorphisms are. Despite this, we can define *derived categories* (or *homotopy categories*) where any of these classes of maps can be treated as if they actually are invertible.

This class will begin with the relevant ideas and results from algebraic topology and homological algebra. Some of the topics from topology include CW approximation and the Whitehead theorem. From algebra we will discuss chain homotopy equivalences and quasi isomorphisms, projective modules, and resolutions. We will then describe the common formal structure behind these examples - a *model category* - and how to use this to define the derived categories and derived functors that are our primary goal.

The topics of this course should be of interest to students in topology, algebra, and combinatorics - anyone who uses techniques from homological algebra (broadly interpreted!). Prerequisites include the prelim sequences in algebra and topology. Algebraic topology I and Homological algebra are useful but certainly not essential.

I will be following several references and one of my main goals for this course is to provide a guided tour that makes the relevant parts of these references more accessible. Standard (though demanding) references for homological algebra and algebraic topology as well as model categories include:

- Dwyer, W. G.; Spaliński, J. Homotopy theories and model categories. *Handbook of algebraic topology*, 73126, *North-Holland, Amsterdam*, 1995.
- Hovey, Mark. Model categories. Mathematical Surveys and Monographs, 63. *American Mathematical Society, Providence, RI*, 1999.
- May, J. P. A concise course in algebraic topology. Chicago Lectures in Mathematics. *University of Chicago Press, Chicago, IL*, 1999.
- Weibel, Charles A. An introduction to homological algebra. Cambridge Studies in Advanced Mathematics, 38. *Cambridge University Press, Cambridge*, 1994.

These are not the place to start without assistance, but they are approachable with guidance.