(1) If E is an equivalence relation on a set X, show that the usual set X/E of equivalence classes can be described as a coequalizer in Set.

(2) Let A be an abelian group and J_A be the category whose objects are the finitely generated subgroups of A with morphisms given by inclusion. Show that A is the colimit of the functor $F\colon J_A \to Ab$ defined by $F(S) = S$.

(3) An object $\emptyset$ of a category D is an *initial object* if for every object d of D there is a unique morphism $\emptyset \to d$ in D. If D has a initial object, show that every functor $F\colon D \to V$ has a limit. Dualize!

(4) Consider the following commutative diagram

$$\begin{tikzcd}
* \arrow[r] \arrow[d] & * \arrow[r] \arrow[d] & * \arrow[d] \\
* \arrow[r] & * \arrow[r] & *
\end{tikzcd}$$

   (a) If both small squares are pullbacks the outside rectangle is also a pullback.

   (b) If the outside rectangle and the right hand square are pullbacks then so is the left hand square.