STABLE HOMOTOPY THEORY
MA 751 : TOPICS IN TOPOLOGY

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This class is an introduction to stable homotopy theory through some of the early papers and books written on these topics.

We often think of the stable homotopy category as *a place where topology acts more like algebra.* This slogan is more than a little vague and, making it more interesting, there are many of ways of making it rigorous. Over the course of the semester we will explore some of these perspectives, but there is one I think is a useful first example:

> Vector spaces have **duals** and if a vector space is finite dimensional it is isomorphic to its double dual.

We can translate the definition of dual to topological spaces, but the only space that has the property of being isomorphic to its double dual is a point - so this definition is not providing anything interesting.

> There is a modification of the definition of dual for spaces where closed smooth manifolds are (stably) homotopy equivalent to their double duals.

This is Atiyah duality and it provides an alternative approach to Poincare duality. This is a useful idea of duality for topological spaces and it is the result of using the definition of dual that comes from vector spaces in the stable homotopy category.

So one interpretation of the slogan above is: *We know duality is useful from experience in algebra. Topology doesn't automatically have it, but it can if we are willing to move to the stable homotopy category.* Many other interpretations of the slogan are given by replacing "duality" by other useful tools from algebra.

The following is a tentative list of books and papers we will read.

- Chapters 1 and 2 of Margolis, H. R. Spectra and the Steenrod algebra. Modules over the Steenrod algebra and the stable homotopy category. North-Holland Mathematical Library, 29. North-Holland Publishing Co., Amsterdam, 1983.
- Chapters 2-4 of Part III of Adams, J. F. Stable homotopy and generalised homology. Chicago Lectures in Mathematics. University of Chicago Press, Chicago, Ill.-London, 1974.
- Lewis, L. Gaunce, Jr. Is there a convenient category of spectra? J. Pure Appl. Algebra 73 (1991), no. 3, 233246.
- Mandell, M. A., May, J. P., Schwede, S., Shipley, B. Model categories of diagram spectra. Proc. London Math. Soc. (3) 82 (2001), no. 2, 441512.

Practical Information:

- It is important to be familiar with ideas from algebraic topology and homological algebra. Prerequisites for this class are MA 551/651/654 (or 655) and MA 561/661/665, but talk to me if you are not sure if the course makes sense for you.
- I expect to allocate the responsibility for presenting parts of these papers and books among the participants in the class.