MA 322 - 09

Assignment 10

1. Let A be a 3×3 matrix whose eigenvalues are 3, $\frac{4}{5}$, and $\frac{3}{5}$ with corresponding eigenvectors $\begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 1 \\ -5 \end{bmatrix}$, and $\begin{bmatrix} -3 \\ -3 \\ 7 \end{bmatrix}$. Let $\vec{x}_0 = \begin{bmatrix} -2 \\ -5 \\ 3 \end{bmatrix}$. If $\vec{x}_{k+1} = A\vec{x}_k$, find an expression for $\vec{x}_k$ starting with the $\vec{x}_0$ above and describe $\vec{x}_k$ as $k \to \infty$.

2. Let F_k be the number of foxes in a particular forest at month k and R_k be the number of rabbits at month k. Suppose

$$F_{k+1} = .5F_k + .4R_k$$
$$R_{k+1} = -pF_k + 1.1R_k$$

In 2(a)-(c) Replace "What is the long term ratio" with "what can you say about the long term behavior?"

The constant p measures the deaths of rabbits due to predation from foxes.

   (a) If $p = .325$ how does the total population of rabbits and foxes change over time? (Is it increasing, constant, or decreasing?) What is the long term ratio of rabbits to foxes?

   (b) If $p = .5$ how does the total population of rabbits and foxes change over time? (Is it increasing, constant, or decreasing?) What is the long term ratio of rabbits to foxes?

   (c) Find a value for p so that the total number of rabbits and foxes does not change over time. What is the ratio of rabbits to foxes in this constant population?

3. Show that $||\vec{v} + \vec{u}||^2 + ||\vec{v} - \vec{u}||^2 = 2||\vec{v}||^2 + 2||\vec{u}||^2$.

4. Let W be a subspace of $\mathbb{R}^n$. Show that if $\vec{u}$ is in W and in $W^\perp$, then $\vec{u} = 0$.

5. Let U be an $m \times n$ matrix where the columns of U form an orthonormal set.

   (a) If $\vec{x}$ and $\vec{y}$ are in $\mathbb{R}^n$, show that $(U\vec{x}) \cdot (U\vec{y}) = \vec{x} \cdot \vec{y}$.

   (b) Show that $||U(\vec{x})|| = ||\vec{x}||$.