## Assignment 5

1. A rental car company has three locations in Lexington. One at the airport, one downtown, and one on Nicholasville road. $95 \%$ of cars rented at the airport are returned there, $2 \%$ are returned downtown and $3 \%$ on Nicholasville road. Of cars rented downtown, $80 \%$ are returned there, $15 \%$ are returned at the airport and $5 \%$ are returned on Nicholasville road. Of cars rented at the Nicholasville road location, $90 \%$ are returned there and the remaining $10 \%$ are retuned at the airport.
(a) What matrix describes the movement of cars between locations?
(b) If the company has a fleet of 90 cars and there are 30 at each location on Monday morning, how many cars are at the airport on Thursday morning?
2. Find a $2 \times 2$ nonzero matrix $B$ (with two different columns) so that $\left[\begin{array}{cc}3 & -6 \\ -1 & 2\end{array}\right] B$ is the zero matrix.
3. Suppose $A$ and $B$ are matrices and $A B$ is defined.
(a) If the second column of $B$ is all zeros what can you say about the second column of $A B$ ?
(b) If the second column of $A B$ is all zero and $B$ has no columns that entirely consists of zeros what can you say about the columns of $A$ ?
4. If $A$ is a $3 \times n$ matrix whose columns span $\mathbb{R}^{3}$, describe how to find a $n \times 3$ matrix $D$ so that $A D=I_{3}$.
5. Show that if $a d-b c=0$ then the equation $\left[\begin{array}{ll}a & b \\ c & d\end{array}\right] \vec{x}=\overrightarrow{0}$ has more than one solution.
6. Let $A=\left[\begin{array}{llll}1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1\end{array}\right]$. Construct a $4 \times 2$ matrix $D$ using only 1 and 0 entries so that $A D=I_{2}$. Is it possible for $C A=I_{4}$ for some $2 \times 4$ matrix $C$ ? (Justify your answers!)
