

# HOMOTOPY THEORY

## MA 752 : TOPICS IN TOPOLOGY

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This class is a continuation of the (unstable) homotopy theory introduced in MA 651 and an introduction to stable homotopy theory.

Understanding topological spaces up to homeomorphism is a really hard question. One of the primary ways we approach it is by trying to classify spaces up to weaker notions of equivalence. If we think of classifying topological spaces up to homeomorphism as identifying the isomorphism classes in the category of topological spaces, we can simplify the problem by moving from the category of topological spaces to a category where more objects are isomorphic. One option is to consider the functor

$$\mathrm{Top} \rightarrow \mathrm{Ho}(\mathrm{Top})$$

which is the identity on spaces and takes a map to its homotopy class. For the first part of the course we will discuss the progress we can make working in the homotopy category. Expected topics include:

- lifting properties and long exact sequences of homotopy groups,
- the Whitehead theorem, and
- the Freudenthal suspension theorem.

For some purposes the homotopy category of spaces still distinguishes between too many spaces. We reach a more manageable category by passing to the stable homotopy category:

$$\mathrm{Top} \rightarrow \mathrm{Ho}(\mathrm{Top}) \rightarrow \mathrm{Ho}(\mathrm{Sp})$$

In the second part of the course we will roughly follow the historical development of the stable category. Expected topics include:

- the stable homotopy category,
- infinite loop spaces,
- generalized cohomology theories, and
- orthogonal spectra

completing the commutative square

$$\begin{array}{ccc} \mathrm{Top} & \longrightarrow & \mathrm{Ho}(\mathrm{Top}) \\ \downarrow & & \downarrow \\ \mathrm{Sp} & \longrightarrow & \mathrm{Ho}(\mathrm{Sp}) \end{array}$$

Practical Information:

- It is important to be familiar with ideas from algebraic topology and homological algebra. Prerequisites for this class are MA 551/651/654 (or 655) and MA 561/661/665, but talk to me if you are not sure if the course makes sense for you.
- There are many statements along the way to the main results that have very self contained proofs. I expect to allocate the responsibility for presenting these among the participants in the class.