A **chain complex** is a sequence of homomorphisms of abelian groups

$$\ldots \longrightarrow C_{n+1} \xrightarrow{\partial_{n+1}} C_n \xrightarrow{\partial_n} \ldots \longrightarrow C_1 \xrightarrow{\partial_1} C_0 \to 0$$

where $\partial_n \circ \partial_{n+1} = 0$ for each $n \in \mathbb{N}$. We define the n^{th} **homology group** of the chain complex to be the quotient $H_n(C) = \mathrm{Ker}\partial_n / \mathrm{Im}\partial_{n+1}$.

Exercise 1. *Compute the homology of the following chain complexes:*

(1) $\ldots \to 0 \to 0 \to \mathbb{Z} \xrightarrow{\mathrm{id}} \mathbb{Z} \to 0$. *Here* $C_1 = C_0 = \mathbb{Z}$ *and all other groups are zero.*

(2) $\ldots \to \mathbb{Z} \xrightarrow{\mathrm{id}} \mathbb{Z} \xrightarrow{0} \mathbb{Z} \xrightarrow{\mathrm{id}} \mathbb{Z} \to 0$. *All groups are* $\mathbb{Z}$ *and the boundary maps alternate between the identity map and the zero map.*

(3) $\ldots \to 0 \to \mathbb{Z}\{U, L\} \xrightarrow{\partial_2} \mathbb{Z}\{a, b, c\} \xrightarrow{\partial_1} \mathbb{Z} \to 0$ *where* $\partial_1(a) = \partial_1(b) = \partial_1(c) = 0$ *and* $\partial_2(U) = \partial_2(L) = a + b - c$.

(4) $\ldots \to 0 \to \mathbb{Z}\{U, L\} \xrightarrow{\partial_2} \mathbb{Z}\{a, b, c\} \xrightarrow{\partial_1} \mathbb{Z}\{u, v, w\} \to 0$ $\partial_1(a) = u - v$, $\partial_1(b) = u - w$ $\partial_1(c) = w - v$, $\partial_2(U) = -a + b + c$, *and* $\partial_2(L) = a - b - c$.