(1) Show that $S^1 \times S^1$ and $S^1 \vee S^1 \vee S^2$ have isomorphic homology groups in all dimensions, but their universal covering spaces do not.

(2) (a) Use the Mayer-Vietoris sequence to compute the homology groups of the space obtained from a torus $S^1 \times S^1$ by attaching a Mobius band via a homeomorphism from the boundary circle of the Mobius band to the circle $S^1 \times \{0\}$ in the torus.

(b) Do the same for the space obtained by attaching a Mobius band to $\mathbb{R}P^2$ via a homeomorphism of its boundary circle to the standard $\mathbb{R}P^1 \subset \mathbb{R}P^2$.

(3) Compute the homology of the following spaces.

(a) The quotient of $S^2$ obtained by identifying north and south poles to a point.

(b) $S^1 \times (S^1 \vee S^1)$.

(c) The quotient space of $S^2$ under the identifications $x \sim -x$ for $x$ in the equator $S^1$.