On this homework you can use the fact that $X \cup C(A)$ is homotopy equivalent to X/A if X and A are CW complexes.

(1) In an exact sequence

$$A \xrightarrow{d} B \xrightarrow{f} C \xrightarrow{g} D \xrightarrow{h} E \xrightarrow{k} F$$

of homomorphisms of abelian groups show the following are equivalent.

   (a) g is an isomorphism.
   (b) f and h are trivial homomorphisms.
   (c) d is surjective and k is injective.

(2) (a) Compute the homology of $S^1 \times S^1$ and $S^1 \vee S^1 \vee S^2$.
   (b) Compute the homology of the universal covers of $S^1 \times S^1$ and $S^1 \vee S^1 \vee S^2$.

(3) (a) Use the Mayer-Vietoris sequence to compute the homology groups of the space obtained from a torus $S^1 \times S^1$ by attaching a Mobius band via a homeomorphism from the boundary circle of the Mobius band to the circle $S^1 \times \{0\}$ in the torus.
   (b) Do the same for the space obtained by attaching a Mobius band to $\mathbb{R}P^2$ via a homeomorphism of its boundary circle to the standard $\mathbb{R}P^1 \subset \mathbb{R}P^2$.

(4) Compute the homology of the following spaces.
   (a) The quotient of S^2 obtained by identifying north and south poles to a point.
   (b) $S^1 \times (S^1 \vee S^1)$.
   (c) The quotient space of S^2 under the identifications $x \sim -x$ for x in the equator S^1.

(5) (a) Show that the quotient map $S^1 \times S^1 \to S^2$ collapsing the subspace $S^1 \vee S^1$ to a point is not nullhomotopic by showing that it induces an isomorphism on H_2.
   (b) Show via covering spaces that any map $S^2 \to S^1 \times S^1$ is nullhomotopic.