MODEL CATEGORIES: HOMOLOGICAL ALGEBRA AND ALGEBRAIC TOPOLOGY
MA 752 : TOPICS IN TOPOLOGY

KATE PONTO
SPRING 2023

Two topological spaces are "the same" if they are homeomorphic. Two groups are "the same" if they are isomorphic. We know from practice that it is actually much more useful to think about homeomorphisms and isomorphisms than to require groups or topological spaces to be identical.

In many situations we want to be even more flexible about what objects we consider to be the same. For example, we many not want to distinguish between homotopy equivalent topological spaces or even spaces that are weakly homotopy equivalent. In algebra we may want to think of chain homotopy equivalent chain complexes or quasi isomorphic chain complexes as the same.

Homotopy equivalences and chain homotopy equivalences have inverses only up to homotopy. Quasi isomorphisms and weak homotopy equivalences have inverses only after taking homology or looking at homotopy groups. None of these are honestly invertible like isomorphisms and homeomorphisms are. Despite this, we can define *derived categories* (or *homotopy categories*) where any of these classes of maps can be treated as if they actually are invertible.

This class will begin with the relevant ideas and results from algebraic topology and homological algebra. Some of the topics from topology include CW approximation and the Whitehead theorem. From algebra we will discuss chain homotopy equivalences and quasi isomorphisms, projective modules, and resolutions. We will then describe the common formal structure behind these examples - a *model category* - and how to use this to define the derived categories and derived functors that are our primary goal.

The topics of this course should be of interest to students in topology, algebra, and combinatorics – anyone who uses techniques from homological algebra (very broadly interpreted!). For a positive expereince in this class students should have taken or be enrolled in MA 651: Topology II and MA 665: Rings and Modules.