The Euler characteristic of a CW complex is

$$\sum_i (-1)^i \# \text{ of } i \text{ cells.}$$

(1) Show that the Euler characteristic of a finite graph $X$ is a topological (really homotopical) invariant of $X$.
(2) Let $F$ be a free group on two free generators $\alpha$ and $\beta$. Let $H$ be the subgroup generated by $\alpha$. Show that $H$ has infinite index in $F$.