Chapter 13: Transformations, Symmetries, and Tilings

Section 2: Patterns and Symmetries
A Symmetry of a Plane Figure

A **symmetry of a plane figure** is any rigid motion of the plane that moves all the points of the figure back to points of the figure.
Reflection Symmetry

A figure has reflection symmetry (also known as line symmetry or bilateral symmetry) if a reflection across some line is a symmetry of the figure.
A figure has **reflection symmetry** (also known as **line symmetry** or **bilateral symmetry**) if a reflection across some line is a symmetry of the figure.

The line of reflection is called a **line of symmetry** or a **mirror line** of the figure.
Rotation Symmetry

A figure has rotation symmetry (also known as turn symmetry) if the figure is superimposed on itself when it is rotated through a certain angle between $0^\circ$ and $360^\circ$. The center of the turn is called the center of rotation. Figures that turn onto themselves after any turn are said to have circular symmetry.
Rotation Symmetry

A figure has rotation symmetry (also known as turn symmetry) if the figure is superimposed on itself when it is rotated through a certain angle between $0^\circ$ and $360^\circ$.

The center of the turn is called the center of rotation.
Rotation Symmetry

A figure has **rotation symmetry** (also known as **turn symmetry**) if the figure is superimposed on itself when it is rotated through a certain angle between $0^\circ$ and $360^\circ$.

The center of the turn is called the **center of rotation**.

Figures that turn onto themselves after *any* turn are said to have **circular symmetry**.
Point Symmetry

A figure has **point symmetry** if it has 180° rotation symmetry about some point $O$. 
Periodic Patterns: Figures with Translation Symmetries

- A periodic pattern is a figure with translation symmetry.
Periodic Patterns: Figures with Translation Symmetries

- A **periodic pattern** is a figure with translation symmetry.

- A **border pattern** has a repeated motif that has been translated in just one direction to create a strip design.
- Periodic Patterns: Figures with Translation Symmetries

- A **periodic pattern** is a figure with translation symmetry.

- A **border pattern** has a repeated motif that has been translated in just one direction to create a strip design.

- A **wallpaper pattern** has a motif translated in two nonparallel directions to create an all-over planar design.
Chapter 13 Transformations, Symmetries, and Tilings

Section 3: Tilings and Escher-like Designs
Tiles and Tiling

A simple closed curve, together with its interior, is a tile. A set of tiles forms a tiling (also known as a tessellation) of a figure if the figure is completely covered by the tiles without overlapping any interior points of the tiles.
Tiles and Tiling

A simple closed curve, together with its interior, is a **tile**. A set of tiles forms a **tiling** (also known as a **tessellation**) of a figure if the figure is completely covered by the tiles without overlapping any interior points of the tiles.
Regular Tilings of the Plane
Regular Tilings of the Plane

A tiling that uses regular polygons as tiles that are joined edge to edge is regular tiling.
Regular Tilings of the Plane

A tiling that uses regular polygons as tiles that are joined edge to edge is regular tiling.

Any arrangement of nonoverlapping polygonal tiles surrounding a common vertex is called a vertex figure.
The Regular Tilings of the Plane

- by equilateral triangles,
- by squares, and
- by regular hexagons.
The Regular Tilings of the Plane

There are exactly three regular tilings of the plane:

(a) by equilateral triangles,
(b) by squares, and
(c) by regular hexagons.
Semiregular Tilings of the Plane
Semiregular Tilings of the Plane

An edge-to-edge tiling of the plane with more than one type of regular polygon and with identical vertex figures is called a semiregular tiling.
Tiling the Plane with Congruent Polygonal Tiles

- Any triangular tile;
- Any quadrilateral tile, convex or not;
- Certain pentagonal tiles (for example, those with two parallel sides);
- Certain hexagonal tiles (for example, those with two opposite parallel sides of the same length).

The plane cannot be tiled by any convex tile with seven or more sides.
Tiling the Plane with Congruent Polygonal Tiles

- The plane can be tiled by

  any triangular tile; any quadrilateral tile, convex or not; certain pentagonal tiles (for example, those with two parallel sides); certain hexagonal tiles (for example, those with two opposite parallel sides of the same length). The plane cannot be tiled by any convex tile with seven or more sides.
Tiling the Plane with Congruent Polygonal Tiles

The plane can be tiled by

- any triangular tile;

any pentagonal tile (for example, those with two parallel sides);

any hexagonal tile (for example, those with two opposite parallel sides of the same length).

The plane cannot be tiled by any convex tile with seven or more sides.
Tiling the Plane with Congruent Polygonal Tiles

The plane can be tiled by

- any triangular tile;
- any quadrilateral tile, convex or not;
- certain pentagonal tiles (for example, those with two parallel sides);
- certain hexagonal tiles (for example, those with two opposite parallel sides of the same length).

The plane cannot be tiled by any convex tile with seven or more sides.
Tiling the Plane with Congruent Polygonal Tiles

The plane can be tiled by

- any triangular tile;
- any quadrilateral tile, convex or not;
- certain pentagonal tiles (for example, those with two parallel sides);
Tiling the Plane with Congruent Polygonal Tiles

The plane can be tiled by

- any triangular tile;
- any quadrilateral tile, convex or not;
- certain pentagonal tiles (for example, those with two parallel sides);
- certain hexagonal tiles (for example, those with two opposite parallel sides of the same length).
Tiling the Plane with Congruent Polygonal Tiles

The plane can be tiled by

- any triangular tile;
- any quadrilateral tile, convex or not;
- certain pentagonal tiles (for example, those with two parallel sides);
- certain hexagonal tiles (for example, those with two opposite parallel sides of the same length).

The plane cannot be tiled by any convex tile with seven or more sides.
Mathematical Reasoning for Elementary Teachers
Edition 5
Long, DeTemple, and Millman
Pearson