If we want to compute the **Instantaneous Rate of Change** at a point \( x = a \), then we usually compute the ARoC between \( x = a \) and \( x = a + h \) where we think of \( h \) as being a small number.

Then

\[
ARoC = \frac{f(a+h) - f(a)}{(a+h) - a} = \frac{f(a+h) - f(a)}{h}
\]

We then let \( h \) get closer and closer to 0, we find a limit.

\[
v(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}
\]

IRoC at \( x \) .... compute

ARoC from \( x \) to \( x + h \) then let \( h \to 0 \)
Ex: A car travels along a straight line with position given by \( f(x) = 9x^2 + 1 \).

a) Find the velocity when \( x = 3 \) seconds.

* Find IROC from \( x = 3 \) to \( x = 3 + h \)

\[
\text{ARoC} = \frac{f(3 + h) - f(3)}{(3 + h) - 3} = \frac{[9(3 + h)^2 + 1] - [9(3)^2 + 1]}{h}
\]

\[
= \frac{9(9 + 6h + h^2) + 1 - 81}{h}
\]

\[
= \frac{81 + 54h + 9h^2 + 1 - 82}{h}
\]

\[
= \frac{54h + 9h^2}{h} = \frac{h(54 + 9h)}{h}
\]

\[
= 54 + 9h
\]

* Now let \( h \to 0 \)

\[
54 + 9(0) = \sqrt{54}
\]
b) Find the velocity when \( x = t \) seconds.

\[
ARoC = \frac{f(t+h) - f(t)}{(t+h) - t} = \frac{[9(t+h)^2 + 1] - [9t^2 + 1]}{h}
\]

\[
= \frac{9(t^2 + 2th + h^2) + 1 - 9t^2 - 1}{h}
\]

\[
= \frac{9t^2 + 18th + 9h^2 + 1 - 9t^2 - 1}{h}
\]

\[
= \frac{h(18t + 9h)}{h}
\]

\[
= 18t + 9h
\]

Let \( h \to 0 \)

\[
18t + 9(0) = 18t
\]

So \( v(t) = 18t \)
Ex: let \( g(k) = k^2 + 4k + 9 \)

a) find the IROC as a function of \( k \).
   
   * find AROC from \( k^\text{th} \) to \( k \)

   \[
   \text{ARoC} = \frac{g(k+h) - g(k)}{(k+h) - k} = \frac{[(k+h)^2 + 4(k+h) + 9] - [k^2 + 4k + 9]}{h}
   \]

   \[
   = \frac{k^2 + 2kh + h^2 + 4k + 4h + 9 - k^2 - 4k - 9}{h}
   \]

   \[
   = \frac{2kh + h^2 + 4h}{h} = h(2k + h + 4)
   \]

   \[
   = 2k + h + 4
   \]

   * IROC let \( h \to 0 \)

   \[
   2k + (0) + 4 = 2k + 4 = \text{IROC}
   \]

b) find the IROC at \( k = 1 \).

   \[
   \text{IROC} = 2(1) + 4 = 6
   \]
the derivative of \( f(x) \) at \( x \),

denoted \( f'(x) \) is

\[
f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}
\]

Other notations:

\[
f'(x) = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x} = \frac{df}{dx}
\]

or if \( y = f(x) \) then

\[
y' = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}
\]
Ex: let \( f(x) = mx + b \). Show \( f'(x) = m \).

\[
f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{m(x+h)+b - (mx+b)}{h}
\]

\[
= \lim_{h \to 0} \frac{mx+mh+b-mx-b}{h}
\]

\[
= \lim_{h \to 0} \frac{mh}{h} = \lim_{h \to 0} m = m
\]

Ex: let \( f(x) = ax^2 + bx + c \). Show \( f'(x) = 2ax + b \).

\[
f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}
\]

\[
= \lim_{h \to 0} \frac{[a(x+h)^2 + b(x+h) + c] - [ax^2 + bx + c]}{h}
\]

\[
= \lim_{h \to 0} \frac{a(x^2 + 2xh + h^2) + bx + bh + c - ax^2 - bx - c}{h}
\]

\[
= \lim_{h \to 0} \frac{2axh + ah^2 + bh}{h}
\]

\[
= \lim_{h \to 0} \frac{2ax + ah + b}{1}
\]

\[
= 2ax + a(0) + b
\]

\[
= 2ax + b
\]