Ex: A 13 ft ladder rests against a wall. If the bottom slides away from the wall at a rate of \( \frac{dx}{dt} = 1 \text{ ft/sec} \), how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 5 ft from the wall?

Solve for \( y \):

\[ 5^2 + y^2 = 13^2 \]
\[ y^2 = 169 - 25 = 144 \]
\[ y = \sqrt{144} = 12 \]

Differentiate \( x^2 + y^2 = 13^2 \) with respect to \( t \):

\[ 2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0 \]
\[ 2(5)(1) + 2(12) \frac{dy}{dt} = 0 \]
\[ 24 \frac{dy}{dt} = -10 \]
\[ \frac{dy}{dt} = \frac{-10}{24} \approx -0.416 \text{ ft/sec} \]
*Note: the answer is negative since the ladder is falling down and distance $y$ is decreasing.

The top of the ladder is falling down the wall at
\[
\frac{10}{24} \text{ or } .416 \text{ ft/sec}
\]
Ex: A cylindrical water tank is being filled at the rate of $4 \text{ ft}^3/\text{min}$. The radius of the tank is $3 \text{ ft}$. How fast is the level of the water in the tank rising when the tank is half full?

Volume of a Cylinder

$V = \pi r^2 h$

Since $r = 3$

$V = \pi (3^2)h = 9\pi h$

differentiate $V = 9\pi h$

$\frac{dv}{dt} = 9\pi \cdot \frac{dh}{dt}$

$4 = 9\pi \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{4}{9\pi} \approx 0.14147 \text{ ft/min}$
Ex: An annual advertising revenue for a newspaper is \( R(x) = 0.4x^2 + 6x + 150 \) thousand dollars when \( x \) is in thousands. Current circulation is 8,000 papers and is increasing by 1,000 papers per year. Two years from now, at what rate will the advertising revenue be increasing?

$$\frac{dx}{dt} = 1$$

Differentiate \( R(x) = 0.4x^2 + 6x + 150 \)

$$\frac{dR}{dt} = (0.8x + 6) \cdot \frac{dx}{dt}$$

In 2 years \( x = 8000 + 1000(2) = 10,000 \) so \( x = 10 \)

Plug in what we know

$$\frac{dR}{dt} = (0.8(10) + 6) \cdot (1)$$

$$= 8 + 6 = 14$$

$14,000 \text{ per year}$
Ex: A stock is increasing in value by $8 per share per year. An investor buys shares at a rate of 20 shares per year. How fast is the value of his stock growing when the stock price is $40 per share and the investor owns 150 shares?

What is the total value?

\[ n = \# \text{ of shares} \]
\[ P = \$ \text{ per share} \]
\[ V = \text{total value} \]

\[ V = nP \]

Take derivative!

\[ \frac{dv}{dt} = \frac{dn}{dt} \cdot P + n \cdot \frac{dp}{dt} \]

\[ \frac{dv}{dt} = (20) \cdot (40) + (150) \cdot (8) \]
\[ = 800 + 1200 \]
\[ = 2,000 \text{ per year} \]