Chapter 6 - Day 1

Extreme Values

The largest value a function attains on an interval is called its global or absolute maximum value.

The smallest value a function attains on an interval is called its global or absolute minimum value.

Both max and min values are called global or absolute extreme values.
Ex: Find the max and min values for the function if they exist.

a) \( f(x) = (x-1)^2 - 3 \)

minimum at \( (1, -3) \)
no maximum

b) \( f(x) = -|x-2| + 3 \)

max at \( (2, 3) \)
no minimum
C) $f(x) = x^2 + 1$ for $x \in [-1, 2]$

- max at $(2, 5)$
- min at $(0, 1)$

We like _continuous_ functions over closed and bounded intervals!

**Recall**: Continuous means no gaps, skips, jumps. You can draw these without picking up your pencil.

An interval is **closed and bounded** if it has finite length and contains its endpoints. **Ex**: $[-2, 5]$ is closed and bounded.
Extreme Value Theorem (EVT): if a function $f$ is continuous on a closed, bounded interval $[a, b]$, then the function $f$ attains a maximum and a minimum value on $[a, b]$.

Example: let $f(x) = \begin{cases} \sqrt{x} & \text{for } x > 0 \\ \sqrt{1-x} & \text{for } x \leq 0 \end{cases}$

Does $f(x)$ have a max and a min on $[-1, 3]$?

EVT says it has both!

- min at $(0, 0)$
- max at $(3, \sqrt{3})$
Ex: Let \( g(x) = \frac{1}{x} \). Does \( g(x) \) have a max and a min on \([-3, 1]\)?

\[ g(x) \] is not continuous
So EVT does not apply.
No max
No min

Ex: Let \( h(x) = x^4 - 2x^2 + 1 \). Does \( h(x) \) have a max and a min on \((-1.25, 1.5)\)?

Open interval - EVT does not apply.
No max
Mins at \((-1, 0)\) and \((1, 0)\)