The first derivative gave us info on increasing/decreasing.

The 2nd derivative gives us info on concavity.

We say $y=f(x)$ is **concave up** on interval $I$ if for points $a<b$ in $I$, the secant line through $(a,f(a))$ and $(b,f(b))$ lies above the graph of $y=f(x)$ for $a<x<b$. 
$y = f(x)$ is **concave down** on interval $I$ if for $a < b$ in $I$, the secant line through $(a, f(a))$ and $(b, f(b))$ lies below the graph of $y = f(x)$ for $a < x < b$.

Also, note the **tangent lines**:

**Concave up**

**Concave down**
Second Derivative Test for Concavity:

- $y = f(x)$ is concave up on $[a, b]$ if and only if $f''(x) \geq 0$ for all $x \in [a, b]$.
- $y = f(x)$ is concave down on $[a, b]$ if and only if $f''(x) \leq 0$ for all $x \in [a, b]$.

A point $(c, f(c))$ on the graph is called a point of inflection if the graph of $y = f(x)$ changes concavity at $x = c$.

**Ex:**

- $f$ concave down on $(-\infty, a)$
- concave up on $(a, \infty)$
- $a$ is an inflection point.
Ex: Sketch a graph of $f(x) = x^4$

$f(x)$ defined everywhere

$f'(x) = 4x^3$ defined everywhere

$f'(x) = 0$ when $x = 0$

$f''(x) = 12x^2$ defined everywhere

$f''(x) = 0$ when $x = 0$

$f''(-1) = 4(-1)^3 = -4$ “−”

$f''(1) = 4(1)^3 = 4$ “+”

$f''(-1) = 12(-1)^2 = 12$ “+”

$f''(1) = 12(1)^2 = 12$ “+”
f(x) increasing on $(0, \infty)$

decreasing on $(-\infty, 0)$

concave up $(-\infty, \infty)$

concave down nowhere

0 is a local min but not an inflection point.

It is helpful to know $f(0) = 0^4 = 0$
so the graph contains the point (0,0)
Ex: Sketch the graph of \( f(x) = xe^{-x} \)

\[
f'(x) = (1)(e^{-x}) + (x)(e^{-x})(-1) \quad \text{*product rule}
= e^{-x} - xe^{-x}
= e^{-x}(1-x)
\]

\( f'(x) = 0 \) when \( x = 1 \)

\[
f''(x) = (e^{-x})(-1)(1-x) + (e^{-x})(-1)
= e^{-x}(x-1) - e^{-x}
= e^{-x}((x-1)-1)
= e^{-x}(x-2)
\]

\( f'''(x) = 0 \) when \( x = 2 \)
$f''(x)$ --- $\uparrow$ inc down local max $\downarrow$ --- $\uparrow$ dec down inflection Pt $\downarrow$ --- $\uparrow$ dec up

$f''(0) = e^0(1-0) = 1 \cdot 1 = 1 \text{ "} + \text{"} $

$f''(1) = (+)(-) = -$  

$f''(\frac{3}{2}) = (+)(-) = -$  

$f''(2) = (+)(-) = -$  

$f''(3) = (+)(-) = -$  

$f''(0) = (+)(-) = -$  

$f''(\frac{3}{2}) = (+)(-) = -$  

$f''(3) = (+)(+) = +$  

$f(1) = 1e^{-1} = \frac{1}{e} \approx .368$

$f(2) = 2e^{-2} = \frac{2}{e^2} \approx .271$