Ex: A farmer builds a rectangular pen with 3 parallel partitions using 600 ft of fence. What dimensions will maximize the total area of the pen?

We know:

\[ 2l + 4w = 600 \]

\[ 2l = 600 - 4w \]

\[ l = 300 - 2w \]

We need to maximize area.

\[ A = l \cdot w \]

\[ = (300 - 2w) w \]

\[ = 300w - 2w^2 \]
because distances are positive...

\[ l \geq 0 \]
\[ 300 - 2w \geq 0 \]
\[ 300 \geq 2w \]
\[ 150 \geq w \]

thus \( w \in [0, 150] \)

Let \( A(w) = 300w - 2w^2 = w(300 - 2w) \)

so \( A'(w) = 300 - 4w \)

\[ 300 - 4w = 0 \]
\[ 300 = 4w \]
\[ 75 = w \]

\( A(w) \) is continuous over closed and bounded interval...
use EVT!
\[ A(0) = 0(300) = 0 \]
\[ A(75) = (75)(150) = 11,250 \leq \text{max} \]
\[ A(150) = (150)(0) = 0 \]

Max area is 11,250 and occurs when

\[ w = 75 \quad \text{and} \quad l = 300 - 2w \]
\[ = 300 - 2(75) \]
\[ = 150 = l \]
Ex: A box is constructed out of 2 different types of metal. The metal for the square top and bottom cost $7 per square foot and the metal for the sides cost $14 per square foot. Find the dimensions that minimize cost if the volume of the box is 30 ft³.

\[
V = s^2h = 30
\]

then \( h = \frac{30}{s^2} \)

We need to minimize cost

\[
C = 7(2s^2) + 14(4sh)
\]

\[
= 14s^2 + 56sh
\]

\[
= 14s^2 + 56s \left( \frac{30}{s^2} \right)
\]

\[
= 14s^2 + 1680s^{-1}
\]
distances are positive...

\[ s \geq 0 \quad h \geq 0 \]

\[ \frac{30}{s^2} \geq 0 \]

two for all \( s > 0 \)

\( s \in (0, \infty) \)

let \( C(s) = 14s^2 + 1680 \) s\(^{-1} \)

then \( C'(s) = 28s - 1680s^{-2} \)

\[ = 28s - \frac{1680}{s^2} \]

\[ = \frac{28s^3 - 1680}{s^2} \]

\( C'(s) = 0 \) when \( 28s^3 - 1680 = 0 \)

\[ 28(s^3 - 60) = 0 \]

\[ s^3 - 60 = 0 \]

\[ s^3 = 60 \]

\[ s = \sqrt[3]{60} \approx 3.9 \]
Since $s$ is in an open interval we need to use the 1st derivative test.

\[ C'(s) \]

\[ \begin{array}{cccc}
0 & \uparrow & 1 & \uparrow \\
\sqrt[3]{60} & & & 4
\end{array} \]

\[ C'(1) = 28(1) - \frac{1680}{1^2} = -1652 \quad "-" \]

\[ C'(4) = 28(4) - \frac{1680}{4^2} = 112 - 105 = 7 \quad "+" \]

Cost is minimized when $s = \sqrt[3]{60} \approx 3.9 \text{ ft}$

and $h = \frac{30}{s^2} = \frac{30}{(\sqrt[3]{60})^2} \approx 1.9 \text{ ft}$