Ex: Consider the graph of $f'(x)$.

a) Where is $f(x)$ increasing?
   - $f'(x)$ is positive
   - $(-\infty, -2) \cup (1, 5)$

b) Where is $f(x)$ decreasing?
   - $f'(x)$ is negative
   - $(-2, 1) \cup (5, \infty)$
c) Where is \( f(x) \) concave up?
\[
f''(x) \text{ is positive - slope of tangent line to } f'(x) \text{ is positive}
\]
\((-1, 3)\)

d) Where is \( f(x) \) concave down?
\[
f''(x) \text{ is negative - slope of tangent line of } f'(x) \text{ is negative.}
\]
\((-\infty, -1) \cup (3, \infty)\)
Ex: An open box is to be made out of a 16 \times 24 \text{ inch} \text{ piece of cardboard by cutting out squares of equal size from the four corners and bending up the sides. Find the dimensions of the box with the largest volume.}
We need to maximize Volume.

\[ V = l \cdot w \cdot h \]
\[ = (24 - 2x)(16 - 2x)(x) \]

distances are \( z \)......

\[ l \geq 0 \]
\[ w \geq 0 \]
\[ h > 0 \]
\[ 24 - 2x \geq 0 \]
\[ 16 - 2x \geq 0 \]
\[ 12 \geq 2x \]
\[ 16 \geq 2x \]
\[ 8 \geq x \]
\[ 8 > x \]

So \( x \in [0, 8] \)

next step... take the derivative!

Let \( V(x) = (24 - 2x)(16 - 2x)(x) \)
\[ = (384 - 80x + 4x^2)(x) \]
\[ = 384x - 80x^2 + 4x^3 \]
then \( V'(x) = 384 - 160x + 12x^2 \)
\[= 12x^2 - 160x + 384\]
\[= 4(3x^2 - 40x + 96)\]

\( V'(x) = 0 \) when \( 3x^2 - 40x + 96 = 0 \)

So \( x = \frac{40 \pm \sqrt{40^2 - 4(3)(96)}}{2(3)} \)
\[x = \frac{40 \pm \sqrt{448}}{6}\]
\[x = \frac{40 \pm 8\sqrt{7}}{6} = \frac{20 \pm 4\sqrt{7}}{3}\]

\( x \approx 10.194 \) and \( 3.139 \)

\( \frac{20 + 4\sqrt{7}}{3} \) is not in interval \([0, 8]\)
$V(x)$ is continuous on a closed and bounded interval... use EVT!

$V(0) = (+)(+)(0) = 0$

$V\left(\frac{20-4\sqrt{7}}{3}\right) = (+)(+)(+) = \max$

$V(8) = (+)(0)(+) = 0$

Max volume occurs when

$$x = \frac{20-4\sqrt{7}}{3} \approx 3.139 \text{ in} = h$$

then $l = 24 - 2x \approx 17.722 \text{ in} = l$

and $w = 16 - 2x \approx 9.722 \text{ in} = w$