Chapter 6 - Day 3

Increasing and Decreasing Functions

$f$ is increasing on an interval $I$ if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ in $I$.

"rising"

$f$ is decreasing on an interval $I$ if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$ in $I$.

"falling"

$f$ increasing

$f$ decreasing
Where is $f$ increasing and decreasing?

$f$ increasing on $[a, b]$ and $[c, d]$
$f$ decreasing on $[b, c]$

Where on this graph is the slope of the tangent line positive? $(a, b)$ and $(c, d)$

Where negative? $(b, c)$
Ex: Suppose \( f(3) = 7 \) and \( f(5) = 12 \). 
\( f \) is increasing on \((3, 5)\) and decreasing on \((-\infty, 3) \cup (5, \infty)\). Are the following possible?

a) \( f(1) = 3 \)  
   not possible

b) \( f(1) = 10 \)  
   possible

c) \( f(4) = 5 \)  
   not possible

d) \( f(6) = 10 \) and \( f(8) = 15 \)  
   not possible

e) \( f(6) = 10 \) and \( f(8) = 6 \)  
   possible

The previous example tells us that:

- if \( f(x) \) is increasing then \( f'(x) > 0 \)
- if \( f(x) \) is decreasing then \( f'(x) < 0 \)
Mean Value Theorem: if $f$ is continuous on $[a, b]$ and differentiable at every point between $a$ and $b$, then there exists some point $x = c$ between $a$ and $b$ such that

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

ARoC from $a$ to $b$ = IRoC at $c$
Ex: let \( Q(t) = t^2 \). Find a value \( A \neq 1 \) such that the average rate of change of \( Q(t) \) from 1 to \( A \) equals the instantaneous rate of change of \( Q(t) \) at \( t = 3 \).

\[
\frac{Q(A) - Q(1)}{A - 1} = Q'(3)
\]

\[
\frac{A^2 - 1^2}{A - 1} = 2(3)
\]

\[
\frac{A^2 - 1}{A - 1} = 6
\]

\[
\frac{(A-1)(A+1)}{A-1} = 6
\]

\[
A+1 = 6
\]

\[
A = 5
\]
Ex: Let \( f(x) = x^3 - x \) on the interval \([-1, 3]\). Find all numbers \( c \) that satisfy the MVT.

\[
\frac{f(3) - f(1)}{3 - (-1)} = f'(c)
\]

\[
\frac{(3^3 - 3) - ((-1)^3 - (-1))}{4} = (3x^2 - 1)|_c
\]

\[
\frac{24}{4} = 3c^2 - 1
\]

\[
6 = 3c^2 - 1
\]

\[
7 = 3c^2
\]

\[
\frac{7}{3} = c^2
\]

\[
c = \pm \sqrt{\frac{7}{3}}
\]

\[-\sqrt{\frac{7}{3}} \text{ not in interval } [-1, 3] \]

\[
C = \sqrt{\frac{7}{3}}
\]