CHINESE RINGS

Description: A set of rings and a metal loop threaded through them that must be removed.

Comments: This set is a "Tavern Puzzle" that I purchased in Berea. It is an example of a combinatorial puzzle. "Stewart Cullin, in his book *Games of the Orient*, records a story that the Chinese Rings puzzle (Lau Kak Ch' A') was invented by the famous Chinese hero, Mung Ming (A.D. 181–234). He apparently gave it to his wife when he went to war so she would have something to keep her busy in his absence. The story relates that she forgot her sorrow while trying to solve the puzzle." (Slocum-Botermans, p. 105.) It was once used as the basis for an absurd stage trick (Gardner, p. 23).

Hints: You will usually discover that at any stage in the process there are at most two rings that can either be put on or taken off the loop. If you represent which rings are on or off the loop at any stage by a sequence of 0's and 1's, you may see a relationship to the binary numbering system. This puzzle is "isomorphic" to the Spin-Out puzzle.

References:

1. Gardner, *Knotted Doughnuts and Other Mathematical Entertainments*, Freeman, 1986, Chapter 2.

2. Slocum and Botermans, *Puzzles Old and New: How to Make and Solve Them*, University of Washington Press, 1987, pp. 104–107.

CHINESE RINGS

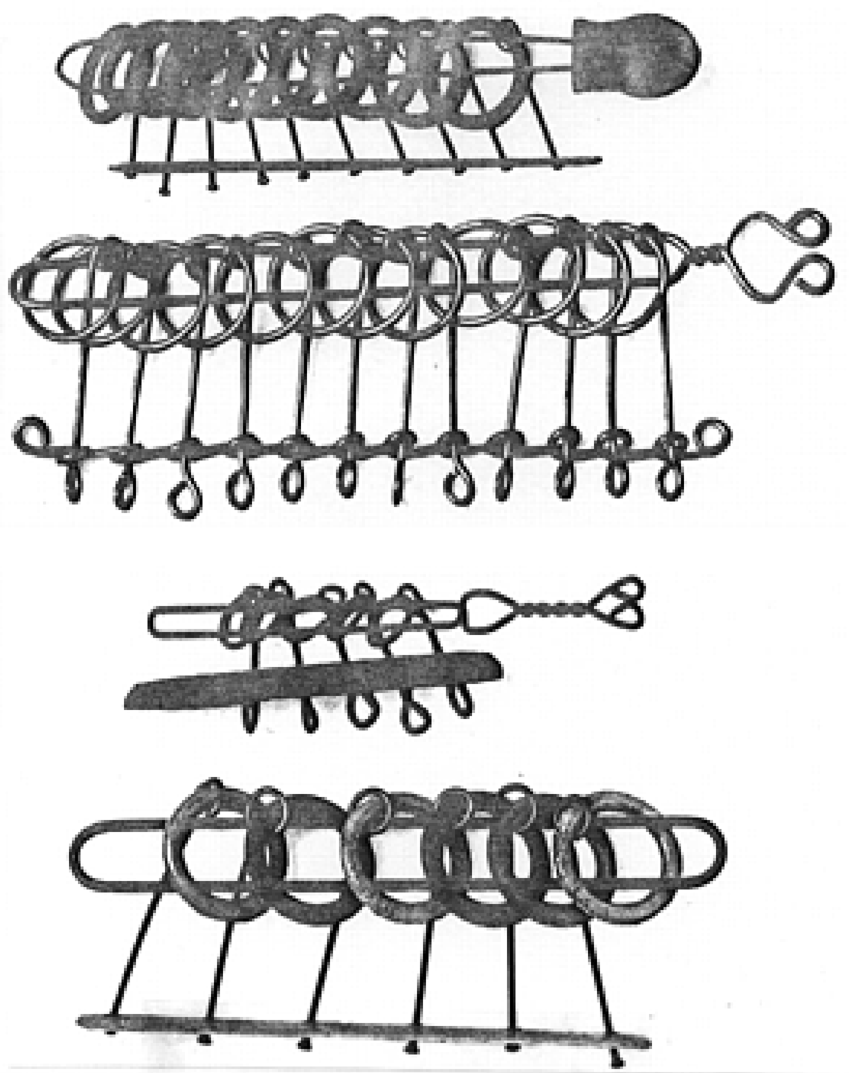


Illustration: Slocum-Botermans, p. 107

CONWAY'S CUBE

Description: The object is to assemble the following pieces into a $5 \times 5 \times 5$ cube: one $2 \times 2 \times 2$ cube, one $2 \times 2 \times 1$ square, three $3 \times 1 \times 1$ rods, and thirteen $4 \times 2 \times 1$ planks.

Comments: This is a contemporary packing puzzle invented by John H. Conway. Another more famous puzzle of this type is the Soma Cube.

Hints: Imagine that the 125 subcubes of the $5 \times 5 \times 5$ cube are colored black and white in a checkered fashion, with the eight corners colored black. How many subcubes of each color are there? What does this imply about the colorings of the constituent pieces? This puzzle lends some insight into the nuances of odd and even quantities.

References: Berlekamp, Conway, and Guy, *Winning Ways for your Mathematical Plays*, Volume 2, Academic Press, 1982, pp. 736, 801.

CONWAY'S CUBE

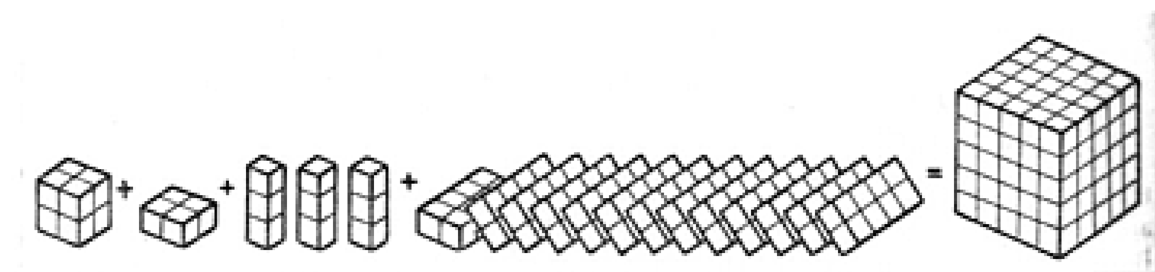


Illustration: Berlekamp-Conway-Guy, p. 736

THE FIFTEEN PUZZLE

Description: 15 numbered squares are arranged in a 4×4 box in the first configuration below, and the object is to slide the squares around until they appear in the consecutive order illustrated in the second configuration below:

1	2	3	4
5	6	7	8
9	10	11	12
13	15	14	

$\longrightarrow$

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

Comments: This is a famous combinatorial puzzle, and can be analyzed using properties of permutations. It was invented by Sam Loyd, America's premiere puzzlist.

"From the mathematical standpoint Loyd's most interesting creation is the famous "14-15" or "Boss" puzzle. This had a surprising revival in the late forties and can still be bought at the toy counters of most five-and-ten-cent stores.... 15 numbered squares are free to slide about within a box. At the beginning of the puzzle the last two numbers are not in serial order. The problem is to slide the squares, without lifting them from the box, until all of them are in serial order, with the vacant space in the lower right-hand corner as before. In the 1870's the 14-15 puzzle had a tremendous vogue both here and abroad and numerous learned articles about it appeared in mathematical journals.

"Loyd offered a prize of \$1,000 for a correct solution to the puzzle. Thousands of people swore they had solved it, but no one could recall his moves well enough to record them and collect the prize." (Gardner, pp. 87–88.)

Hints: There is a good reason why no one collected the prize.

References:

1. Gardner, *The Scientific American Book of Mathematical Puzzles and Diversions*, Simon and Schuster, 1959, Chapter 9.
2. Slocum and Botermans, *Puzzles Old and New: How to Make and Solve Them*, University of Washington Press, 1987, pp. 127–128.

THE FIFTEEN PUZZLE

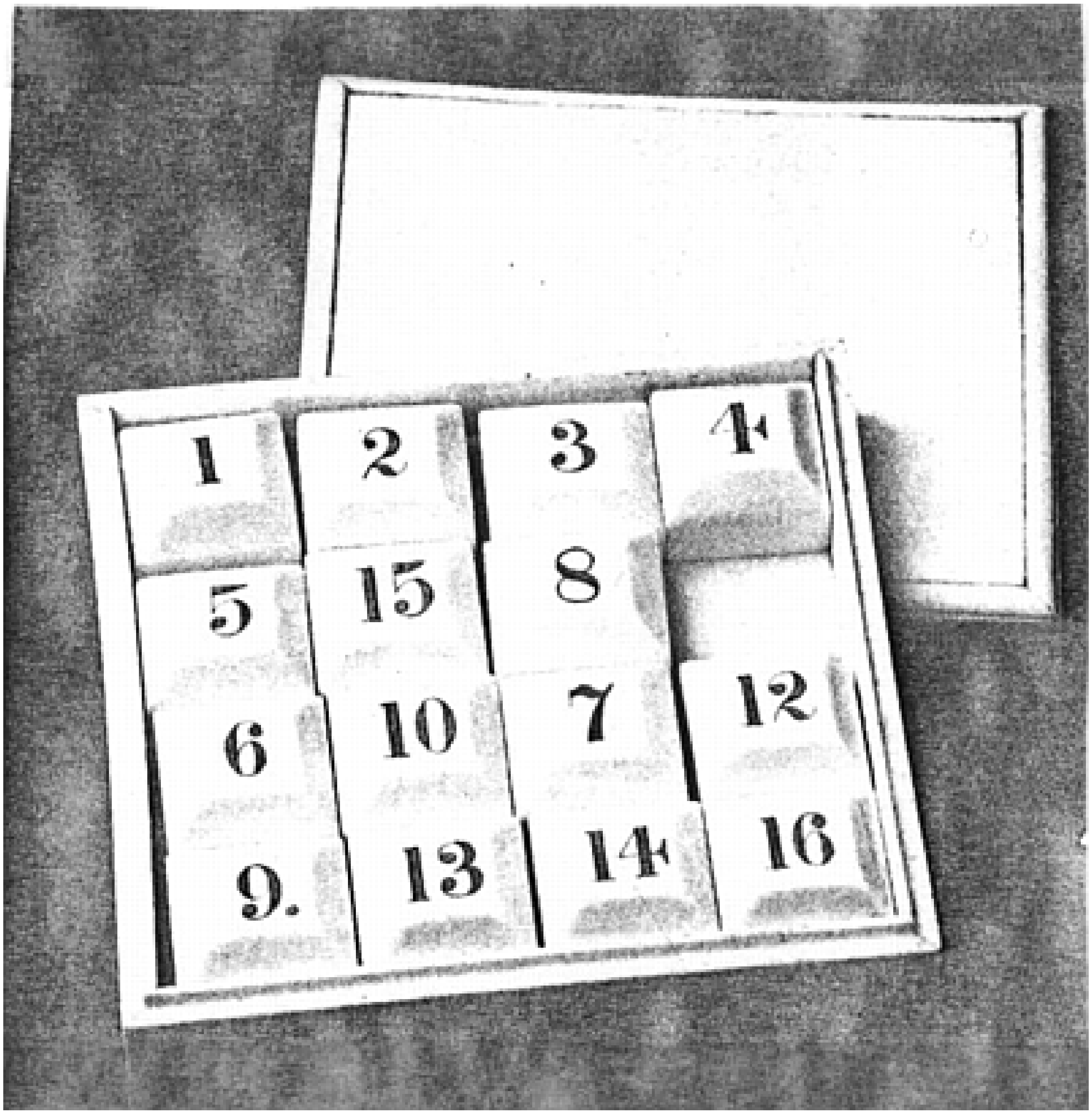


Illustration: Slocum-Botermans, p. 128

TOWER OF HANOI

Description: This well-known puzzle consists of three pegs, with a stack of disks of decreasing size on one peg. The object is to move the disks one at a time from peg to peg, ultimately transferring the entire stack onto another peg. The condition is that at no time is a larger disk to be placed on a smaller disk.

Comments: "The familiar Tower of Hanoi is a combinatorial puzzle invented by the French mathematician Edouard Lucas and sold as a toy in 1883." (Gardner, p. 57.) It is also known as the "Tower of Brahma" because of the fanciful legend Lucas created for it.

Hints: Try to solve the puzzle with three disks, and then four disks. Think about how the solution to the three-disk puzzle can be used as a part of the solution to the four-disk puzzle, and so on. This is an excellent example of a puzzle with a solution that can be described "recursively," and there are also strong connections to the binary numbering system.

References:

1. Berlekamp, Conway, and Guy, *Winning Ways for your Mathematical Plays*, Volume 2, Academic Press, 1982, pp. 753.
2. Gardner, *The Scientific American Book of Mathematical Puzzles and Diversions*, Simon and Schuster, 1959, Chapter 6.
3. Van Delft and Botermans, *Creative Puzzles of the World*, Abrams, 1978, p. 175.

TOWER OF HANOI

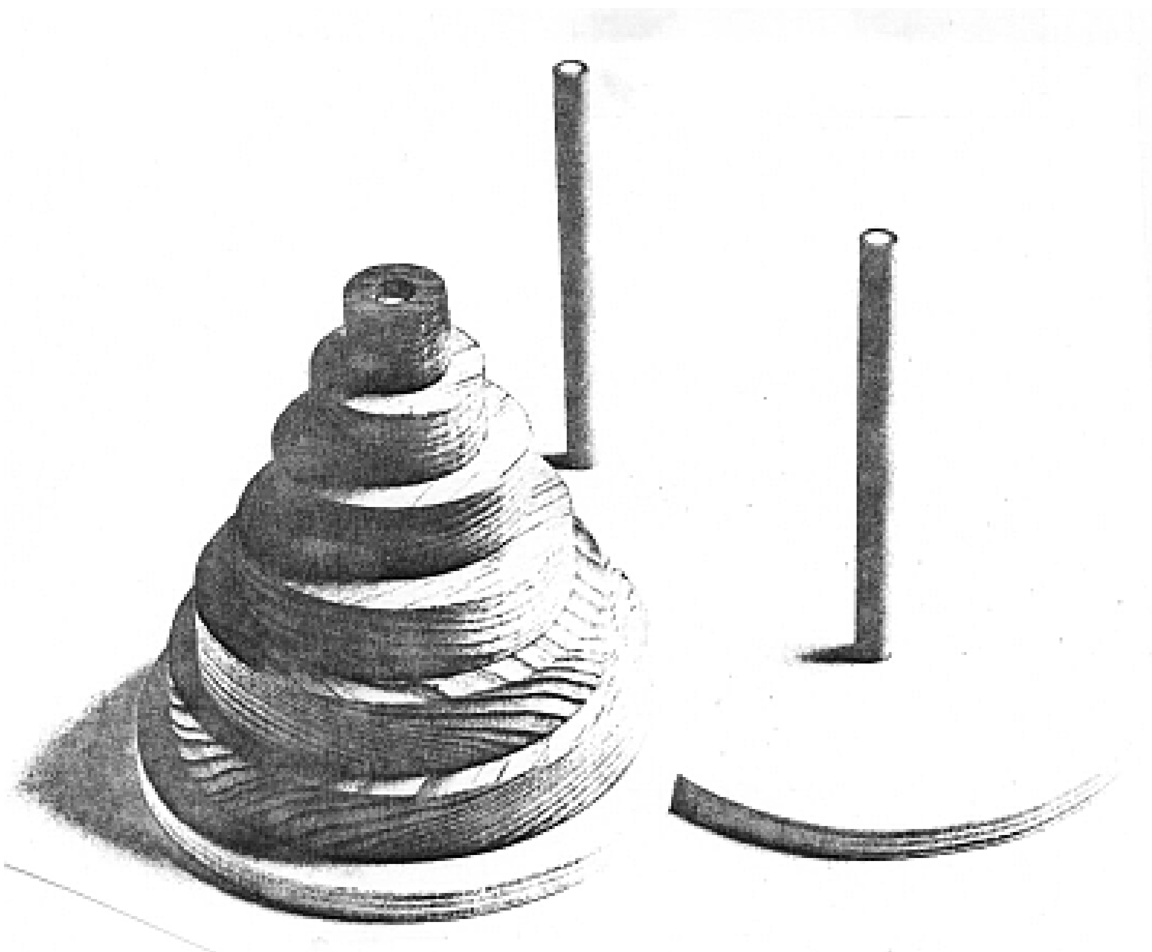


Illustration: Van Delft-Botermans, p. 175

HEXSTICKS

Description: A "burr" puzzle consisting of 12 notched hexagonal rods. The object is to disassemble and then reassemble the puzzle.

Comments: This beautiful burr puzzle was designed in the early 1970's by Stewart Coffin, "the most outstanding designer and builder of interlocking puzzles the puzzle world has ever seen." (Slocum-Botermans, pp. 79, 84.) It is an example of a geometrical dissection puzzle with striking three-dimensional symmetry, and is one of my favorites.

Hints: In one method of assembly, the final three pieces must be inserted simultaneously. But "[T]here are three distinctly different solutions to this puzzle, which can be defined by the arrangement of the three-notch pieces." (Coffin, p. 116.)

References:

1. Coffin, *The Puzzling World of Polyhedral Dissections,* Oxford, 1991, Chapter 13.
2. Slocum and Botermans, *Puzzles Old and New: How to Make and Solve Them*, University of Washington Press, 1987, pp. 62–85.

HEXSTICKS

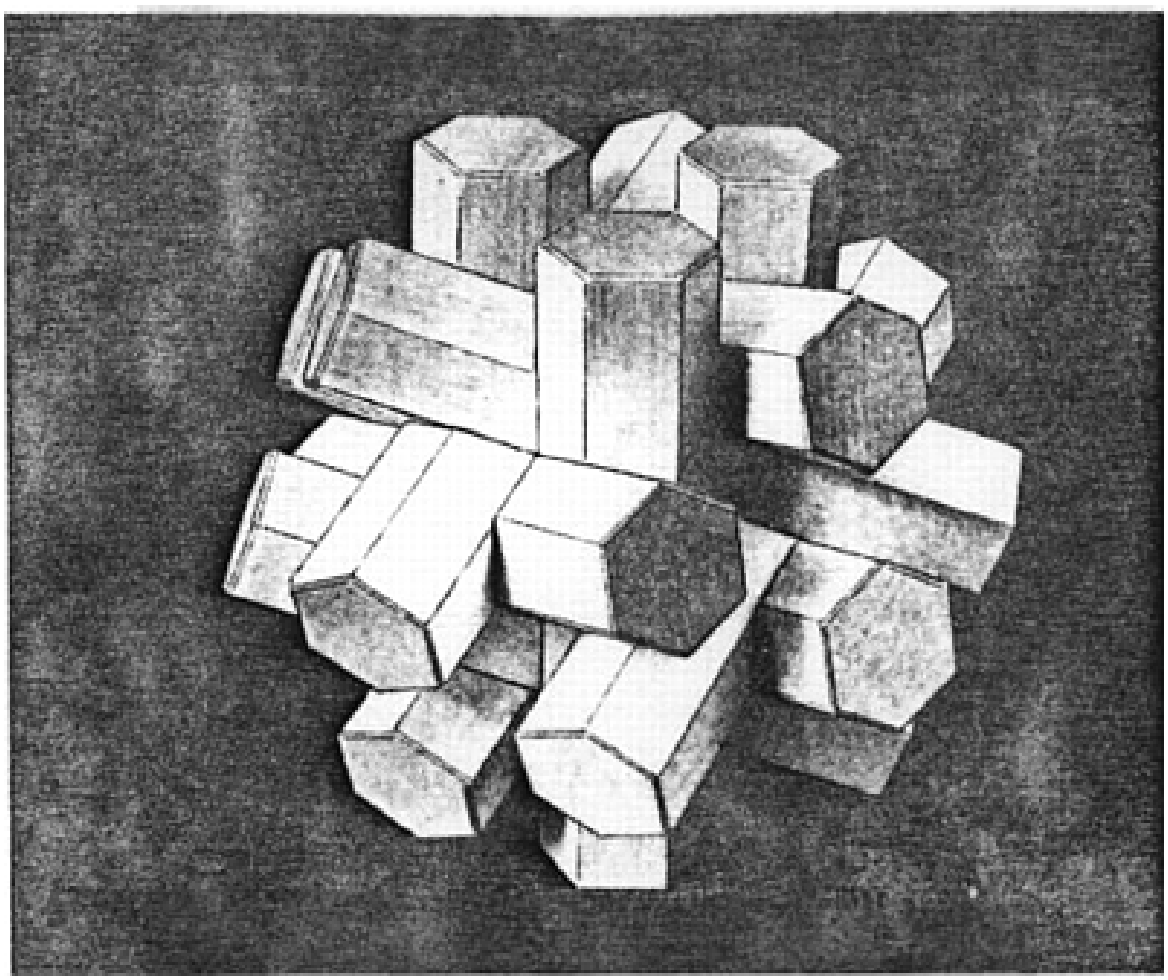


Illustration: Coffin, p. 117

THE HORSESHOE PUZZLE

Description: The object is to remove the ring from the pair of horseshoes, and then put it back on again.

Comments: One form of the horseshoe puzzle was manufactured under the name "Ring of the Nibelungs" in Germany in 1915. It is clear that if horseshoes and the ring were made out of rubber, the ring could be removed. We can say that the ring and the horseshoes are not "topologically linked." But they are not flexible, so the problem is to show that they are not "geometrically linked" either. My children can take this apart in a half-second!

Hints: You do not have to force anything to get the ring off. When manipulated properly, it just falls off.

References: Slocum and Botermans, *Puzzles Old and New: How to Make and Solve Them*, University of Washington Press, 1987, p. 99.

THE HORSESHOE PUZZLE

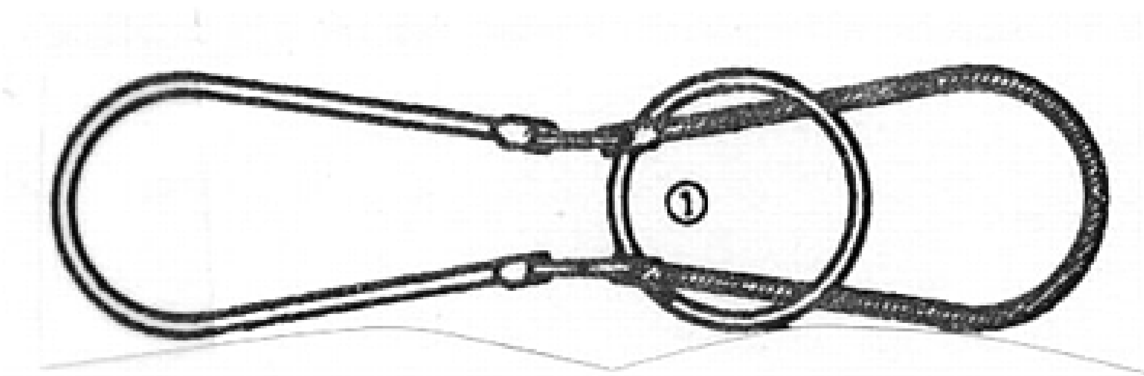


Illustration: Slocum-Botermans, p. 99

PYRAMID PUZZLE

Description: Assemble these two identical pieces to form a perfect tetrahedron (a three-dimensional solid with four equilateral triangular faces—a pyramid with a triangular base).

Comments: I do not know the origin of this puzzle, but it has been around a long time. I think I got a plastic one out of a cereal box when I was a kid. This is a very simple example of a geometrical dissection puzzle, but a classic nonetheless.

Hints: This is one time when an unconscious desire for symmetry might lead you down a blind alley.

References: Slocum and Botermans, *Puzzles Old and New: How to Make and Solve Them*, University of Washington Press, 1987, p. 46.

PYRAMID PUZZLE

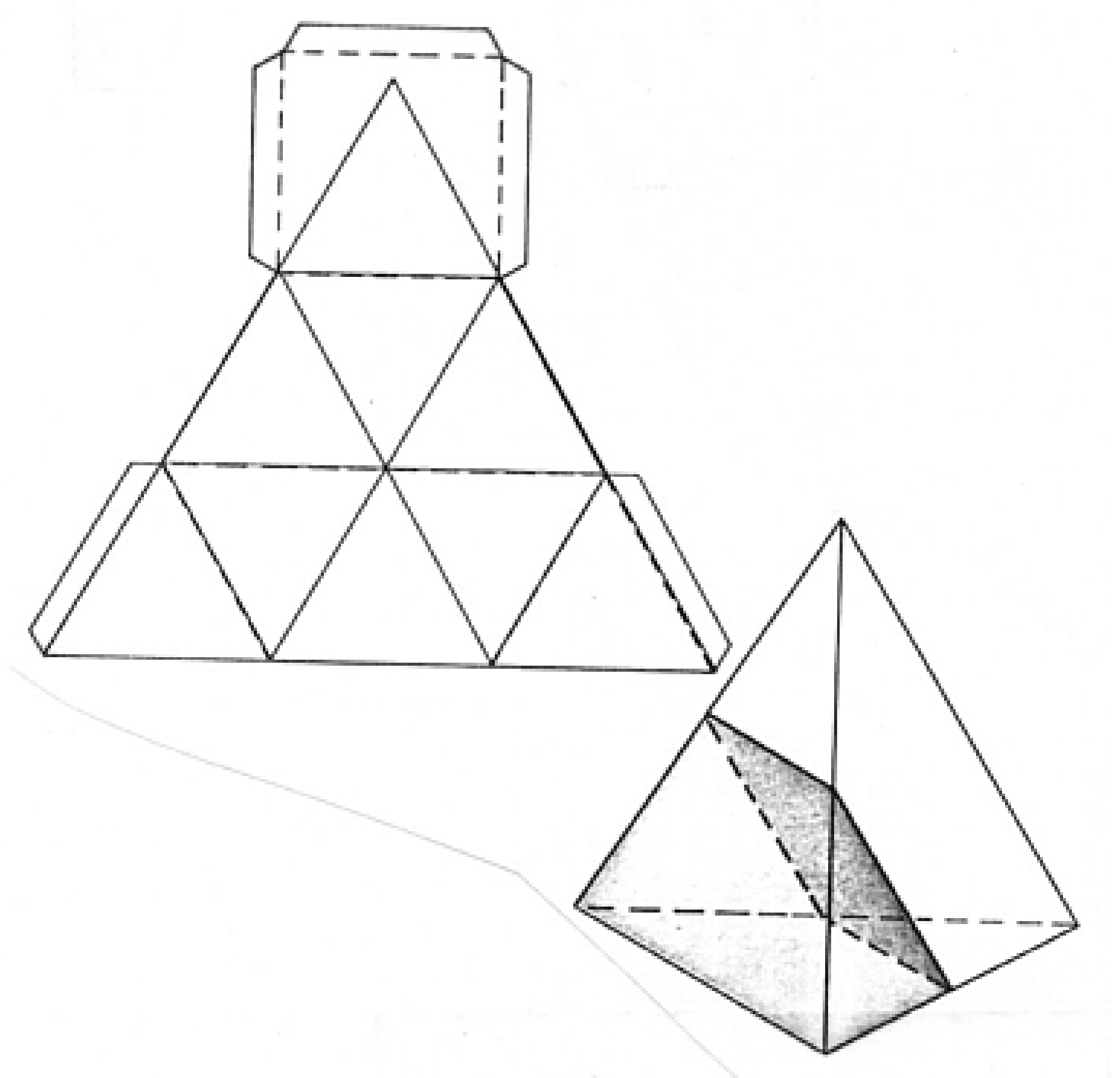


Illustration: Slocum-Botermans, p. 46

ROPE PUZZLE

Description: This well-known puzzle requires two pieces of rope, each about five feet in length, and two people. One end of the first rope is loosely tied to the first person's right wrist, and the other end to the first person's left wrist. The rope should not, however, be so loosely tied that it can slip off over the hands. The second rope is likewise tied to the second person's wrists, except that it is passed through the loop of the first rope so that the two ropes are linked together. The object is for the two people to unlink their ropes without untying or cutting them.

Comments: I do not know the origin of this topological puzzle; it has been around a long time. I first saw it in cub scouts.

Hints: I have seen people tie themselves up in knots trying to do this, while others have thoughtfully stared at the ropes for a few minutes and then successfully carried out the required manipulations. Perhaps you should start by convincing yourself that the two ropes are not really linked.

References: Van Delft and Botermans, *Creative Puzzles of the World*, Abrams, 1978, p. 120.

ROPE PUZZLE

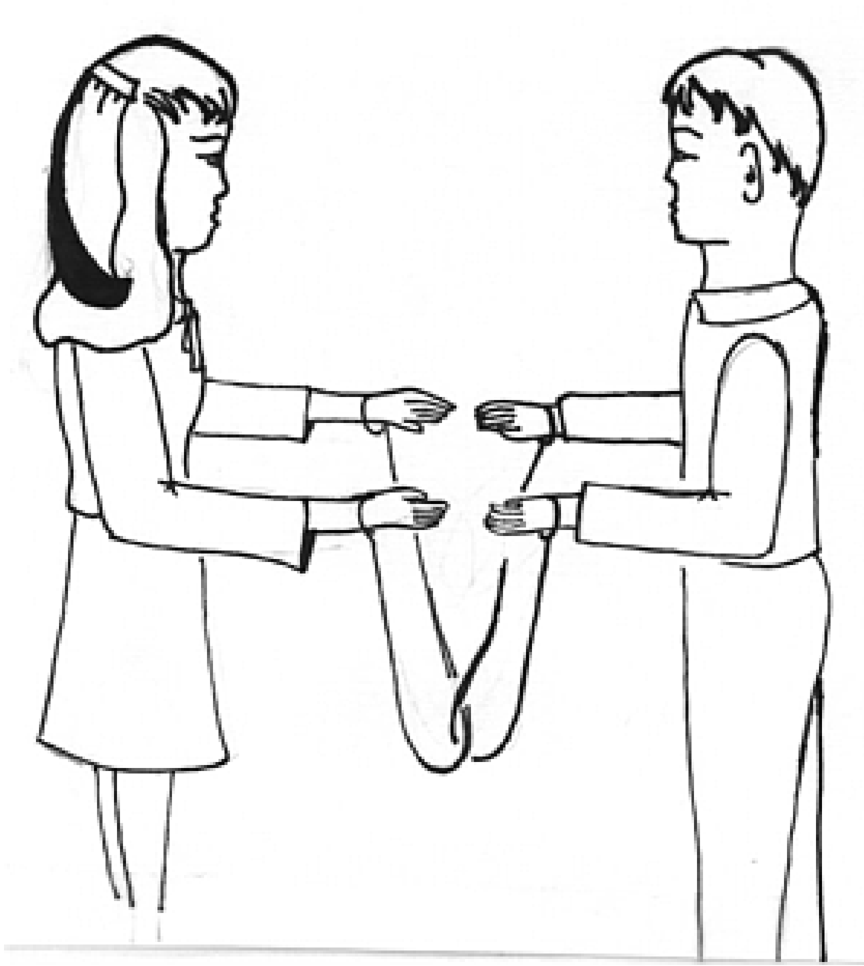


Illustration: Gwenyth Lee

THE SOMA CUBE

Description: The object is to assemble the seven given pieces into a $3 \times 3 \times 3$ cube.

Comments: This is an example of a packing puzzle which I find quite aesthetically pleasing. "Piet Hein conceived of the Soma cube during a lecture on quantum physics by Werner Heisenberg. While the noted German physicist was speaking of a space sliced into cubes, Piet Hein's supple imagination caught a fleeting glimpse of the following curious geometrical theorem. If you take all the irregular shapes that can be formed by combining no more than four cubes, all the same size and joined at their faces, these shapes can be put together to form a larger cube" (Gardner, *Second Scientific American Book...*, p. 66).

Hints: Try positioning the more complicated shapes first. Don't despair; there are over 240 distinct ways of solving the puzzle.

References:

1. Gardner, *Knotted Doughnuts and Other Mathematical Entertainments*, Freeman, 1986, Chapter 3.

2. Gardner, *The Second Scientific American Book of Mathematical Puzzles and Diversions*, Simon and Schuster, 1961, Chapter 6.

THE SOMA CUBE

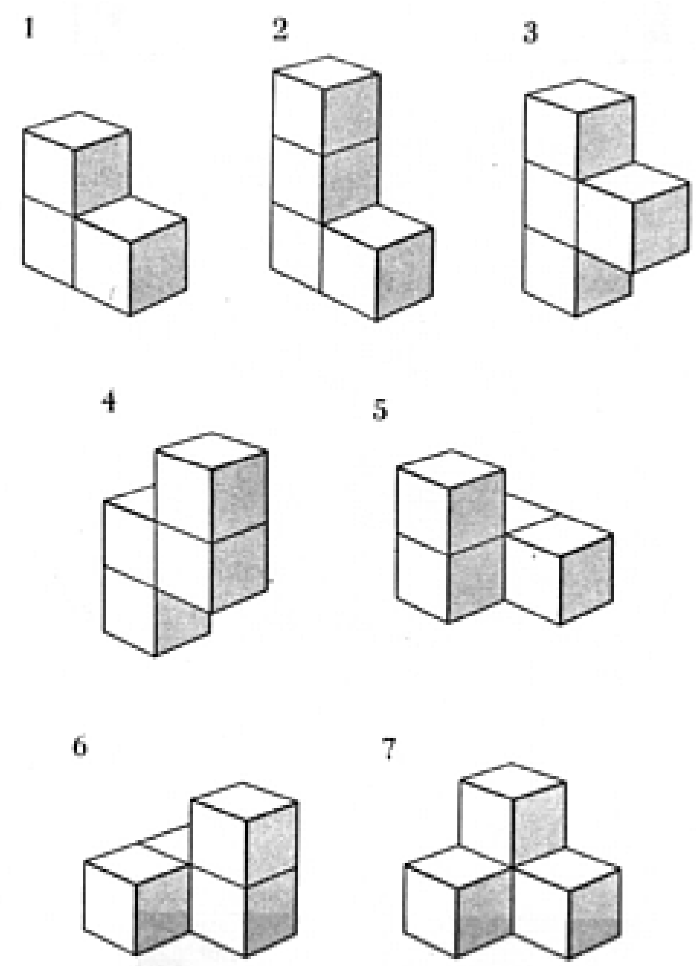


Illustration: Gardner, *Second Scientific American Book...*, p. 67

SPIN-OUT

Description: A slide with attached notched knobs is to be removed from a sleeve by turning the knobs appropriately.

Comments: This is a contemporary combinatorial puzzle produced by Binary Arts.

Hints: You will usually discover that at any stage in the process, there are at most two dials that can be turned. If you represent the orientations of the dials at any stage by a sequence of 0's and 1's, you may see a relationship to the binary numbering system. This puzzle is "isomorphic" to the Chinese Rings puzzle.

References: Gardner, *Knotted Doughnuts and Other Mathematical Entertainments*, Freeman, 1986, Chapter 2.

STAR PUZZLE

Description: On the one hand, this is a geometric dissection puzzle: take apart and re-assemble the three-dimensional "star." Note that the constituent pieces of the wooden one and the plastic one are slightly different—the plastic one is easier. On the other hand, this is a packing puzzle: Take a number of (assembled) plastic stars and try to pack them tightly together to fill space.

Comments: "Someone, somewhere, perhaps in the mid-19th century, made the marvellous discovery that the ends of the diagonal burr sticks can be bevelled to produce a puzzle that, when assembled, is the first stellation of the rhombic dodecahedron. According to puzzle collector and historian Jerry Slocum, a puzzle of this sort was sold as early as 1875. The only patent on it that the author is aware of is Swiss Patent No. 245 402 to Iffland in 1946.

"The word *intriguing* is used frequently throughout this book to describe various polyhedral dissections, but none can outshine the brilliance of this simple dissection.... From one point of view, it may be regarded as a diagonal burr puzzle in which bevelling ends of the pieces produces a totally unexpected and beautiful new shape. From another point of view, it is a surprising dissection of the stellated rhombic dodecahedron into six identical pieces that amazingly assemble and interlock! It has more interesting properties, too: when viewed along one of its fourfold axes of symmetry it is square, while along one of its threefold axes it is the Star of David. And perhaps most surprisingly of all, it is a space-filling solid." (Coffin, pp. 75–76)

Hints: Persevere—this is not an extremely hard puzzle, but assembling the wooden one requires some dexterity (and sometimes an extra pair of hands!).

References: Coffin, *The Puzzling World of Polyhedral Dissections*, Oxford, 1991, Chapter 7–8.

STAR PUZZLE

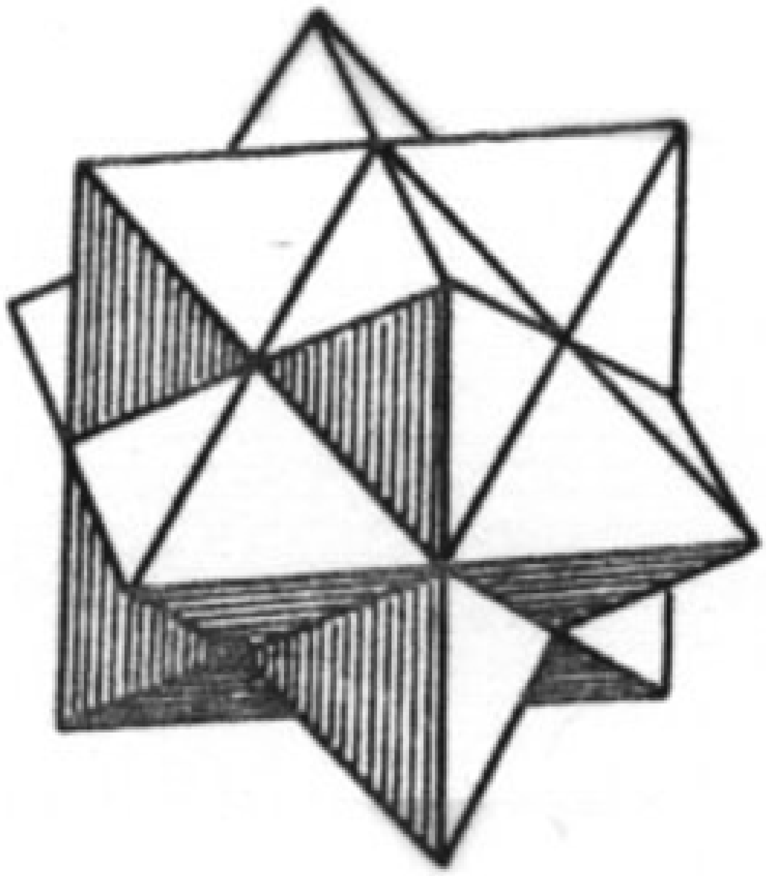


Illustration: Coffin, p. 76

TENSEGRITY STRUCTURES

Description: Informally, a tensegrity structure is a pattern of rods held in place by elastic bands in such a way that no two rods touch each other. Can you construct the tensegrity tetrahedron, consisting of six rods?

Comments: Perhaps you have seen the tensegrity sculpture outside the Hirshhorn museum in Washington, D.C. These structures arise in the study of stress and rigidity, and some have been used in the design of buildings. "They were first conceived of by the sculptor Kenneth Snelson and then popularized by [Buckminster] Fuller, his teacher." (Kappraff, p. 310.)

Hints: It should not be too difficult to copy the display model by examining it carefully. In fact, any polyhedron can be used as the basis of a tensegrity structure, and there are many more possibilities beyond these.

References:

1. Kappraff, *Connections: The Geometric Bridge Between Art and Science*, McGraw-Hill, 1991, Chapter 8.
2. Manual for Tensegritoy, manufactured by Tensegrity Systems Corporation, Barrytown, NY, 12507, 1987.

TENSEGRITY STRUCTURES

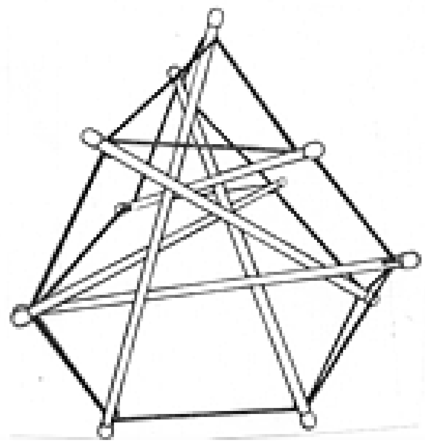
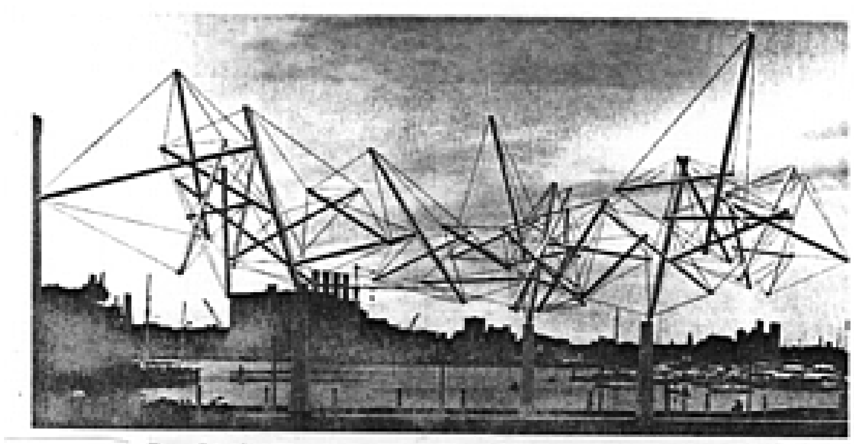


Illustration: Tensegritoy instruction manual, p. 6



Easy Landing, a tensegrity structure by Kenneth Snelson.

Illustration: Kappraff, p. 310

REGULAR TILINGS

Description: A regular tiling is a tiling of the plane consisting of multiple copies of a single regular polygon, meeting edge to edge. How many can you construct?

Comments: While these tilings can be easily found by trial and error, and all are quite familiar, they do give some insight into the geometry of regular polygons.

Hints: There are only three regular tilings.

References: Kappraff, *Connections: The Geometric Bridge Between Art and Science*, McGraw-Hill, 1991, Chapter 5.

REGULAR TILINGS

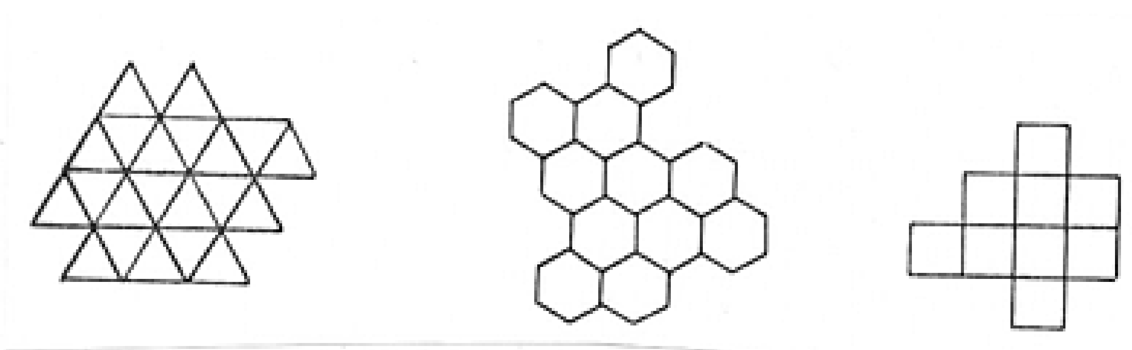


Illustration: Kappraff, p. 179

SEMIREGULAR TILINGS

Description: A semiregular tiling is a tiling of the plane consisting of at least two different types of regular polygons. In addition, the pattern of polygons meeting at a common juncture point (or vertex) must be the same for each point. How many can you construct?

Comments: Some of these tilings are not immediately obvious. You can calculate or eliminate some possibilities by making a table of the measures of the interior angles of regular polygons, since the sum of these angles for the polygons meeting at a common vertex must be 360 degrees.

Hints: There are only eight semiregular tilings.

References: Kappraff, *Connections: The Geometric Bridge Between Art and Science*, McGraw-Hill, 1991, Chapter 5.

SEMIREGULAR TILINGS

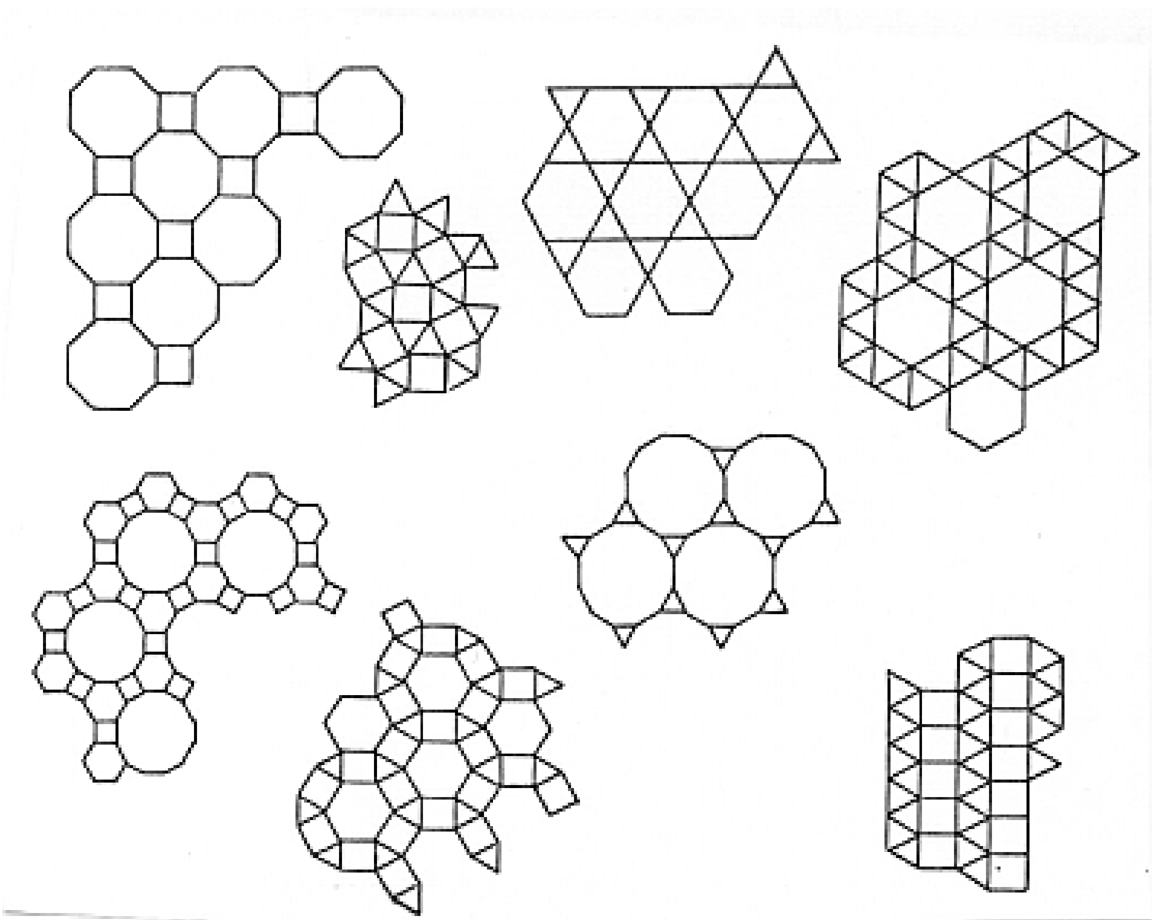


Illustration: Kappraff, p. 179

PLATONIC SOLIDS

Description: A Platonic solid has the property that each face is an identical regular polygon, and that the same number of polygons meet at each corner. Can you construct them all?

Comments: These are ancient objects of study. Plato (naturally!) is reputed to have investigated them, and they are discussed in Euclid's *Elements*, but they may be quite older.

Hints: There are only five Platonic solids:

1. Tetrahedron. Four equilateral triangles, three meeting at each corner.
2. Cube. Six squares, three meeting at each corner.
3. Octahedron. Eight equilateral triangles, four meeting at each corner.
4. Dodecahedron. Twelve regular pentagons, three meeting at each corner.
5. Icosahedron. Twenty equilateral triangles, five meeting at each corner.

References:

1. Holden, *Shapes, Space, and Symmetry*, Dover, 1971.
2. Wells, *The Penguin Dictionary of Curious and Interesting Geometry*, Penguin, 1991, pp. 187–188.

PLATONIC SOLIDS

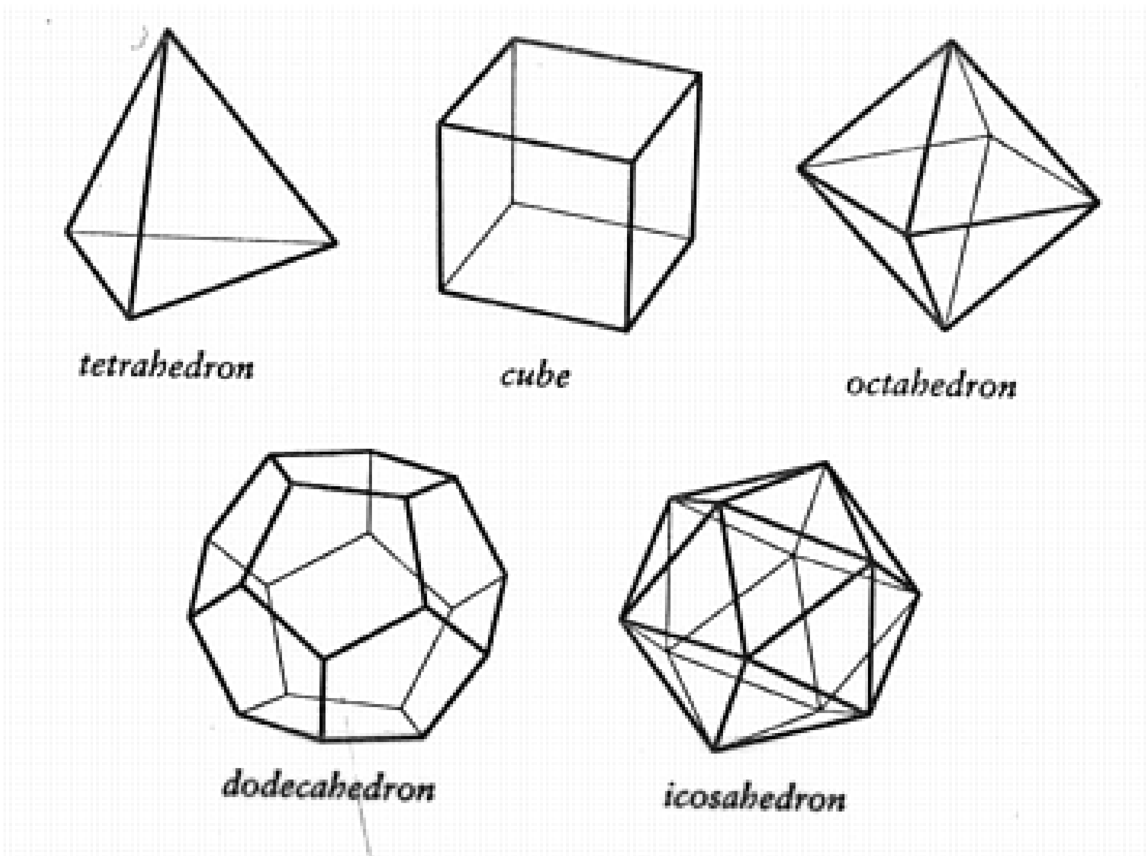


Illustration: Wells, p. 188

SEMIREGULAR SOLIDS

Description: A semiregular solid has the property that each face is a regular polygon, but that there are at least two different kinds of faces. In addition, the same pattern of faces must meet at each corner. How many can you construct?

Comments: According to Heron and Pappus, Archimedes wrote about these solids, but unfortunately the work has been lost. These are beautiful objects with very appealing symmetry.

Hints: Take two copies of any regular polygon and use them as the top and bottom of a "drum" ringed with squares. This is a prism. Now remove the squares and use a strip of equilateral triangles to form the sides of the drum. This is an antiprism. There is an infinite number of prisms and antiprisms—one for each choice of base polygon. Apart from these, there are exactly thirteen semiregular solids, also called Archimedean solids. One of them is made with hexagons and pentagons, and is the template for a soccer ball, as well as a representation of a recently discovered form of carbon, C_{60}, known as Buckminsterfullerene.

References:

1. Holden, *Shapes, Space, and Symmetry*, Dover, 1971.
2. Pearce and Pearce, *Polyhedra Primer*, Dale Seymour, 1978, pp. 55–66.
3. Wells, *The Penguin Dictionary of Curious and Interesting Geometry*, Penguin, 1991, pp. 6–8.

SEMIREGULAR SOLIDS

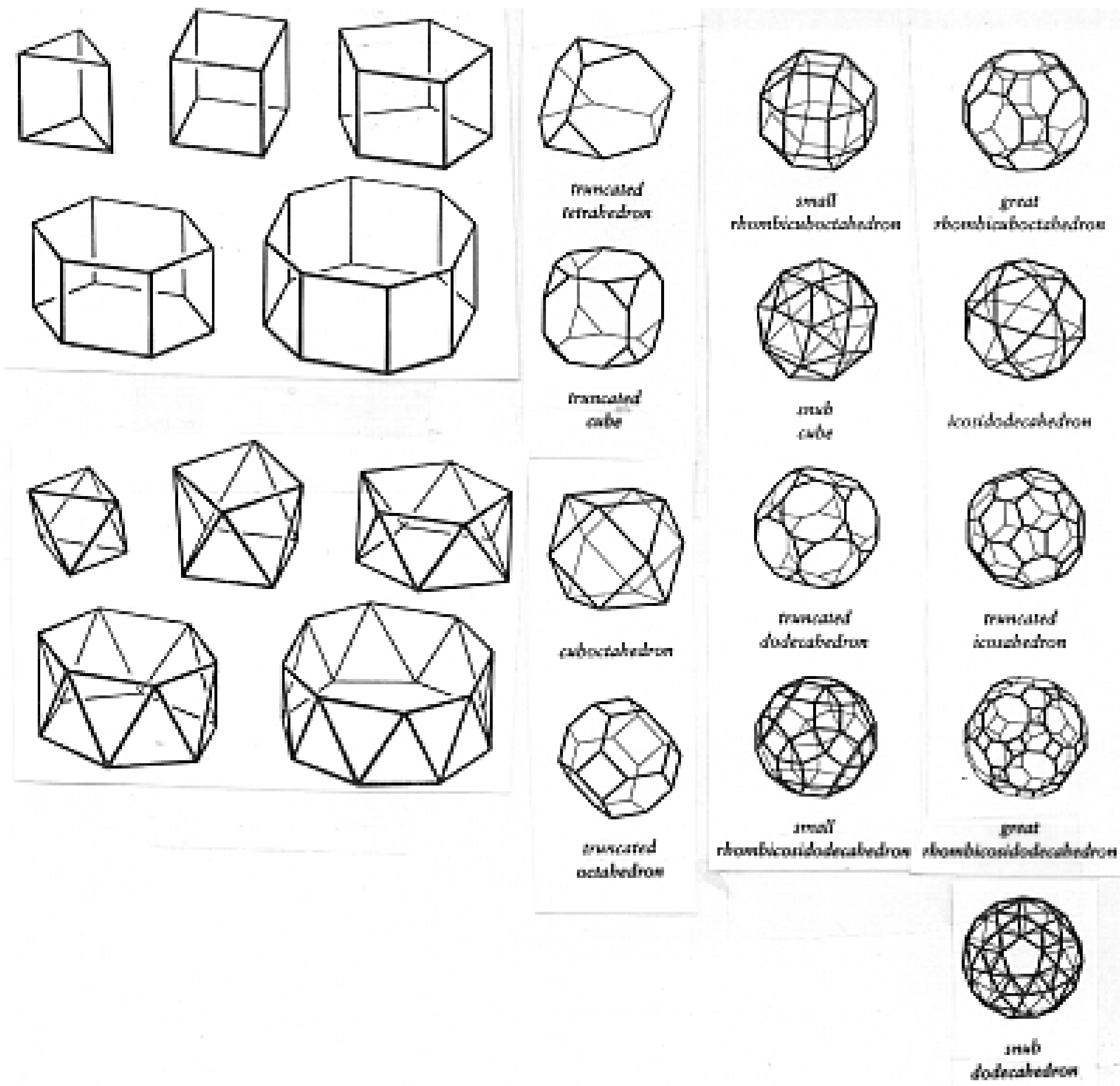


Illustration: Wells, pp. 6–7, Pearce-Pearce, p. 66

THE DELTAHEDRA

Description: A deltahedron is a polyhedron made solely from equilateral triangles. There is no requirement that the same number of triangles meet at every corner, but the object must be convex (i.e., it should have no indentations) and two triangles that meet along an edge must not lie in the same plane. How many can you construct?

Comments: It is conjectured that the name comes from the fact that the Greek letter Δ looks like a triangle.

Hints: Try to first understand why every deltahedron has an even number of faces. Experiment to see if you can determine the minimum and maximum number of faces. Curiously, one potential deltahedron is "missing"!

References:

1. Holden, *Shapes, Space, and Symmetry*, Dover, 1971, p. 3.

2. Pearce and Pearce, *Polyhedra Primer*, Dale Seymour, 1978, p. 67.

THE DELTAHEDRA

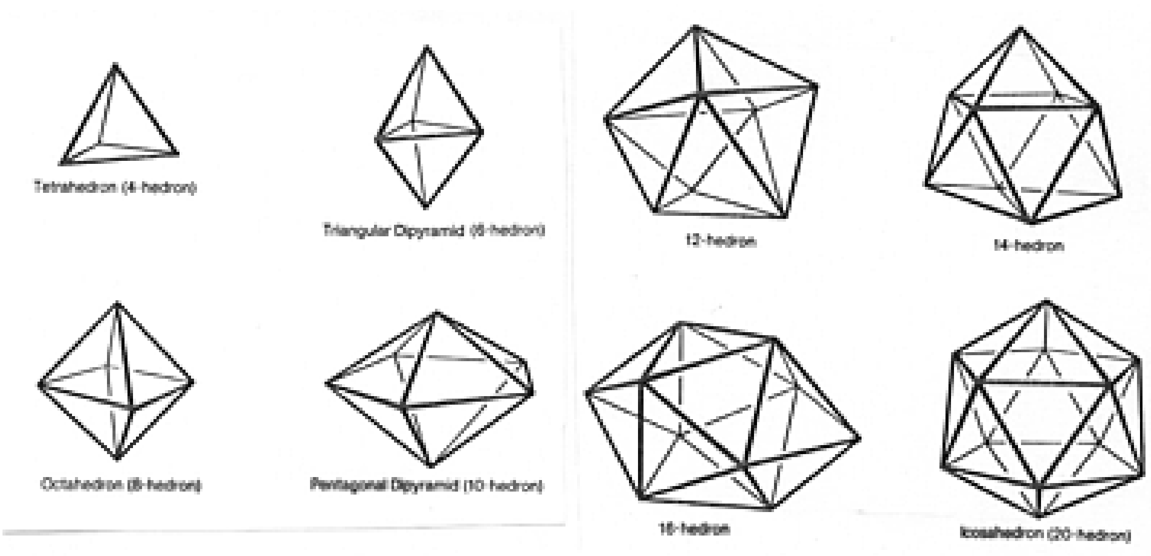


Illustration: Pearce-Pearce, p. 67