

Consider the equation $2x + 1 = 5x - 4$. You'd probably begin the finding-a-solution process by adding or subtracting some quantity from both sides?

Why? That's a rhetorical question for right now, one that we'll answer in a while. Let's back and define our terms. What is an equation? What is a solution to an equation? What is the solution set for an equation?

1. Recall that an *algebraic expression* is one involving constants, variables, and operation symbols. For example, $2x + 1$, $5x - 4$, $x^2 - 2xy + \pi$ are all algebraic expressions.
2. Assigning values to each variable in algebraic expression (and doing some computation) leads to *evaluation* of the algebraic expression.
3. An equation is just a pair of algebraic expressions separated by an equal sign.

Now suppose you have some equation E involving just one variable, say t . We'll write it this way: $A(t) = B(t)$, where A and B are algebraic expressions.

What do we mean by a solution to an equation of the form E ?

What do we mean by the *solution set* to E above?

Let's return to $2x + 1 = 5x - 4$. Suppose we add 4 to each side of the equation. We get a new equation: $2x + 5 = 4x$. How is the first equation related to the second?

Suppose E is any equation, involving just one variable (to make things easy for the time being). Let E_1 be the equation that results from adding the same thing to both sides, say a constant. How is the solution set to E related to that of E_1 ? Think about how you would explain your answer to that last question.

In similar fashion, multiplying both sides of an equation by the same non-0 constant, results in a new equation which has what in common with original equation?

Two equations are *equivalent* if they have the same solution set.

To guarantee that such an addition results in an equivalent equation, are we restricted to adding the **constant** to both sides?

Same question with “multiplying” in place of “adding”.

Same question if we use “square” in place of ”adding”.

Let’s run through a procedure and solve the equation $2x + 1 = 5x - 4$ and discuss it.

Now let’s decide together if we’d like to work in Z_{12} , the integers mod 12. Less sophisticated folk work in Z_{12} . In elementary math even, usually referring to it as “clock arithmetic (algebra) ”. A solution is an element of $Z_{12} = \{0, 1, \dots, 10, 11\}$. I think we might even be able to make up a story problem involving clocks which is modelled by the equation above.

Let’s work together on finding the solution set in Z_{12} .

Let’s compare with finding a solution in the real numbers.