Seminar 3: Equation-solving III

A+S 101-003: High school mathematics from a more advanced point of view

Goals of today's discussion:

1. Reflect on why the standard algorithm for solving linear equations in one variable is valid? To help guide you: *equivalence* of equations played a big role. Why is the last equation (of form "$x = c$") the (only) solution to the linear equation we began with?

2. Which operations on an (arbitrary) equation lead to an equivalent equation? Discussion.

3. Review and reflect on solving linear equations in one variable in other algebraic systems: linear congruences. Compare with linear equations in the real numbers.

4. Discuss quadratic equations. Why can't we solve a quadratic by "isolating the variable"?

5. Begin developing an algorithm to solve quadratic equations.

Problem 1 Let E be an equation. Which of the following operations to E results in an equivalent equation?

1. Adding the same constant to both sides of the equation.

2. Adding the same algebraic expression to both sides, assuming that involves only the variables in that algebraic expression are in E also.

3. Adding the same algebraic expression to both sides of E, assuming the algebraic expression contains variables not in E.

4. Squaring both sides of E.

5. Cubing both sides of E.

6. Multiplying both sides of E by the same constant.

7. Multiplying both sides of E by the same algebraic expression, assuming the algebraic expression involves only variables contained in E.

Challenge 2. A challenge put forth by Dr. Jones: how can the problems above be re-stated so that it could be presented to a middle-school or high-school student?

Problem 3. They're working around the clock at HumbleSoftware, in shifts of 9 hours. They began at midnight (think of it as 0 hour). They only have one powerful computer, so only one person can work at a time. Suppose there will be five shifts, one right after the other. One hour after the end of the fifth shift, the big bosses will be meeting. Mr. Big, the office boss of bosses, whose not so sure about this whole thing, wants to know what his time clock will read when it's time to go to the meeting. You can score big points with Mr. Big by letting him know when to head over to the meeting.

Try to model the problem via a linear equation involving one variable. Will you solve it in the real numbers or in $Z_1 2$?

Problem 4. Mass confusion again this week at HumbleSoftware. Mr. Big can't remember the length of the shifts he assigned– only that the shifts were of same length and the shift length in hours is a whole number less than 12. He knows the clock reads 12 after the six shifts. Model the situation with an equation and solve. What can you tell the strange Mr. Big?

Quadratic equations in the real numbers

The algorithm for solving a linear equation with one variable calls for *isolating the variable.* Consider the equation

$$x^2 = -2x + 5.$$

Question. Is isolating the variable helpful here? Explain.

Well, if we can't quickly solve $x^2 = -2x + 5$, let's work on one a little easier– maybe by solving an easier case, we can get some ideas for $x^2 = -2x + 5$. We'll imagine we haven't seen quadratic equations before, the situation your students will be in at some stage of their education.

Problem. Consider the equation $(x + 1)^2 = 4$. How might we solve it?

Problem. Now think about $x^2 = -2x + 5$.

We continue the discussion (assuming time permits)– our goal is an algorithm for quadratic equations in the real numbers.