Seminar 6: *Quadratics; inequalities and absolute values; long-division, remainders and factoring*

A+S 101-003: High school mathematics from a more advanced point of view

1. We'll continue with quadratic equations today: the connection between finding roots, graphs, and factoring.
2. We may see up to two presentations today.
3. As time permits: Why does factoring a function $f(x)$ help find roots of $f(x)$?
4. Division stuff: numbers and polynomials

Quick review

Problem 1. Let $q(x) = x^2$. How can you find the minimum value of $f(x)$? Can you do it without graphing, or even visualization?(That is, argue you have the correct min using number facts.)

Problem 2. Let $f(x) = (x-2)^2 + 5$. How can you find the minimum value of $f(x)$? Can you do it without graphing, or even visualization?

Problem 3. Let $h(x) = |x-4| + 5$. How can you find the minimum value of $f(x)$? Can you do it without graphing, or even visualization?

Problem 4. Make up real-world maximization (or minimization) problem modeled by a quadratic function.

Discussion problem 5. What's a *root* of a quadratic function? What connection does a root a quadratic have with its graph? Can you explain that connection?

Problem 6. Find the roots of

1. $f(x) = x^2 - 3x + 2$
2. $g(x) = -x^2 + 4x + 1$

Problem 7. What connection does the factorization of a quadratic have to with its roots as a function? For example, let $q(x) = x^2 - 3x + 2$. Factor $q(x)$. Determine its roots.

Problem 8. Suppose we have factored a quadratic $q(x) = (x - c)(x - d)$. What are the roots of $q(x)$? Explain. How can you be sure that we haven't missed any roots? Before you explain, work on the next problem.

Problem 9. Let's work in Z_{12}. Let $f(x) = (x - 2)(x - 4)$ Are the roots of $f(x)$ just 2 and 4?

Problem 10. What property of real numbers (not possessed by Z_{12}) guarantees that factoring completely determines the root set of a quadratic?

Absolute values and inequalities too: because they are so important in high school math

Let's recall the 50 cent definition of absolute value: $|x| = x$, if $x \geq 0$; otherwise, $|x| = -x$ (when $x < 0$).

Problem 11. Explain how to find solutions to $|x| = 2$?

To $|x - 1| = 2$?

To $|2x - 1| - 2 = 0$?

Problem 12. Now you want to find solutions to $|x| > 1$. How many solutions are there? Can you provide a graphical description of the solution set?

Problem 13. Solve $|2x - 1| \geq 3$.

Problem 14. Connect absolute values and the idea of distance on the number line.

Problem 15. How would you solve $|x^2 - 4| \geq 0$?

Loose ends: should we go here?

Please look this over this week. I think some discussion of long-division, divisibility (in numbers and polynomials) might be interesting (and useful ultimately to you).

The famous Division Algorithm (so obvious it's "hard to see"..but having so many uses).

Let a, b be integers, with $b \neq 0$. Then there exist **unique** q (the quotient) and r (the remainder) , with r satisfying $0 \leq r < b$, such that $a = bq + r$.

Let $a = 1001$ and $b = 21$. Find q and r guaranteed by the Division Algorithm.

Unless you're good at mental arithmetic, you might do a long-division.

One consequence of the uniqueness aspect of the Division Algorithm: an integer is either even or odd and it can't be both! (Of course everyone knows that anyway..so of course it's a consequence of a detestable theorem)

For that matter, why does the long-division algorithm (on integers) work anyway? Simple explanation? Ever try to find one? (I didn't..but now I'm sorry I didn't– because I have to teach the validity of that algorithm to pre-service elementary teachers, which makes me feel sort of phony but relieved to find out it's explainable in simple terms.)

Theorem. Let $p(x)$ be a polynomial of degree n. Then r is a root of $p(x)$ if and only if $p(x) = (x - r)q(x)$, where $q(x)$ is a polynomial of degree $n - 1$.

What does this theorem say about the number of roots of a polynomial of degree n?