Seminar 7: *Proposed new topics discussion; polynomials, roots, factoring*
A+S 101-003: High school mathematics from a more advanced point of view

1. Volunteer to take class notes.
2. Last time Brad presented on probability.
3. We discussed maximization and minimization of quadratics, using an intuition and numeric skills. For example: Let $h(x) = |x-4|+5$. How can you find the minimum value of $f(x)$? Can you do it without graphing, or even visualization?
4. Following up on Dr. Jones' suggestion to find ways to link up formal mathematics with real-world problems, we discussed the following problem: A rectangular enclosure is to be built with available materials, 100' of fence. To save materials (so that more area can be enclosed with the limited resources), a side of a barn will be used to enclose one side. How can we maximize the area of the enclosure?
5. Brad presentation made mention of the *binomial theorem* and *Pascal's triangle*. We considered the possibility that these (connected) topics might be treated in more depth.
6. We discussed the connection between the factorization of a (quadratic) polynomial and its roots. We took note of the fact that factoring is an effective way to find roots primarily **because** the product of two non-0 real numbers is non-0. We observed that that property fails in Z_{12} and as a consequence a polynomial such as $p(x) = (x-2)(x-3)$ has more than two (its degree) roots.

Where to go from here?

1. We've discussed equation-solving (solving an equation by finding a sequence of equivalent equations, each with more transparent solution set).
2. We've discussed linear and quadratic equations in detail; we've derived the quadratic formula (twice!)
3. We've discussed the connections solution sets, graphs (and their x-intercepts) and factoring of polynomials.
4. We've compared equation-solving and factoring in two domains: the reals and in Z_{12}.
5. We've begun presentations: presenters are asked to choose a topic of interest to themselves.

Potential directions

Here are some topics that have been of interest to participants. Topics overlap.

1. Probability, finite probability
2. Binomial theorem, Pascal's triangle
3. Number theory
4. Trigonometry from a more advanced point of view
5. Generalizing from "equation-solving" to "inequality solving".

Here's a preview of a potential topic, counting permutations and combinations and connections with the binomial theorem.

What does the binomial theorem say and how it is connected to Pascal's triangle?

Binomial Theorem. Suppose x and y are variables taking values potentially in the real numbers. Then $(x+y)^n = \sum_{i=0}^{i=n} \binom{n}{i} x^i y^{n-i}$.

We have some undefined terms: what is "$\binom{n}{i}$" ? Note: in middle and high school texts (and some college ones as well), $C(n,r)$ is used instead of $\binom{n}{i}$. By the way, it's read as "n choose r".

Show that $2^n = \sum_{i=0}^{i=n} \binom{n}{i}$. (Brad essentially did this in his talk.)

Prove that the number of subsets of an n-element set is 2^n.

Calculate some fairly simple but interesting finite probabilities using the ideas here.

Discuss connections with Pascal's triangle.

Understanding things, proving things, being able to explain things, finding interesting examples...