Seminar 9: *Continuation on binomial theorem, Pascal's triangle, and finite probabilities*
A+S 101-003: High school mathematics from a more advanced point of view

1. We will need a volunteer to take class notes.
2. Last time we introduced the concept "n choose r" in an informal way, by asking: what's the probability that if we flip a coin n times that exactly r heads show up?
   - We came to the conclusion that the answer was a number over 2^n, where the number in the numerator is the number of ways we can choose r objects from a set of n objects. Let's denote that number by $C(n, r$ (read it as "n choose r").
   - We worked out several examples, including that $C(4, 2) = 6$.
   - We pretty much agreed that determining $C(10, 5)$ by counting up the number of "5-subsets" of a 10 element set would be tedious.
   - So it might be worth our time to come up with a formula for $C(n, r)$.
   - If computing probabilities on coin tosses wasn't reason enough to find a formula for $C(n, r)$, we found another reason (or we were close..). We determined that for a whole number n, that $(x + 1)^n = \sum_{i=0}^{i=n} C(n, i)x^i$.
   - "Discovery" was used more than in other seminars. Perhaps most interestingly, toward the end of the seminar, Dr. Jones capped the discovery by asking a series of telling questions, beginning with: "what did you understand"? "what didn't you understand?" "what questions do you have?", and as the replies came, Dr. Jones engaged the students with further questions. By some kind of wildcat magic, the connection between the dice throwing and the "binomial coefficients" came to be for the room.
   - How can we find a formula for $C(n, r)$? Let's try to find a formula via a guided discovery.

Guide: permutations, $P(n, r)$ and so on

Our goal: find a formula for $C(n, r)$. We need some background- let's prepare a background via example.

Consider the letters a, b, c. List all ways you can you re-arrange (or, *permute*) those letters?

How many permutations are there of a three element set?

Suppose you have 4 letters. How many ways can the n letters be permuted?

Find a formula for the number of permutations of n letters.

Suppose we have 4 students, a, b, c, d. How many different ways are there to choose a president, vice-prez, sec'y, and treasurer (assuming all positions are filled by different students)?

Suppose we have 4 students, a, b, c, d. How many different ways are there to choose a president and a vice-prez from among them (assuming prez and vice prez are different students)?

Suppose we have 40 students. How many ways to choose a prez and vice prez?

How many three letter words are there in English, each such word having no repeating letter?

Suppose we have an alphabet of n letters. We want to count all r-letter words, each such word having no repeating letters (here $n \geq r \geq 0$). Denote this number by $P(n, r)$. Find a formula for $P(n, r)$.

How are $P(n, r)$ and $C(n, r)$ related?

To help answer this question, let's think about $P(4, 2)$ and $C(4, 2)$.