Seminar 11: *Discussion IV on binomial theorem, Pascal's triangle, and finite probabilities*

A+S 101-003: High school mathematics from a more advanced point of view

1. We will need a volunteer to take class notes.
2. We discussed taking up a new topic and agreed upon a Calculus topic.
3. Last time we consolidated our understanding of relationships among $P(n,r)$, $C(n,r)$ and the number of permutations of r objects. We reviewed the binomial theorem, making sure to understand the connection between the coefficients and the number of elements of certain subsets.
4. We considered the problem: how many distinct subsets does an n-element set have? We came up, via discovery, with 2^n in two very different ways. First, it was observed that the number of subsets is equal to $\binom{n}{0} + \binom{n}{1} + \ldots + \binom{n}{n} = \sum_{r=0}^{r=n} \binom{n}{r}$.

   We realized that that $(1+1)^n = \sum_{r=0}^{r=n} \binom{n}{r}$, by the Binomial Theorem.

   Also, we set up a one-to-one correspondence between the subsets of $\{1, \ldots, n\}$ and the set n-tuples $\{(a_1, \ldots, a_n) : a_i \in \{0,1\}\}$. This last set isn't hard to "count"- it has 2^n elements.
5. Today we'll review and discuss Pascal's Triangle in more detail.

Before we begin with Pascal's Triangle, let's do an interesting review application of binomial coefficients, a problem "transferrable" to high school.

Problem An urn contains 4 red marbles and 5 blue marbles. You choose 3 marbles at random. What's the probability that the marbles chosen are the same color?

Pascal via discovery

You've got 10 marbles: 9 are black, one is magenta. You'll choose 4 marbles. There are no doubt $\binom{10}{4}$ ways to do so.

Break things down so that you can conceptually explain the equation $\binom{10}{4} = \binom{9}{4} + \binom{9}{3}$ (to a high school student).

Stare at the equation above. Generalize it. Explain why the equation you wrote down (which involves binomial coefficients) is valid.