

A&S 100 – 007  
Statistics for Teachers

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# 1 Lecture 1

## 1.1 Collecting Data

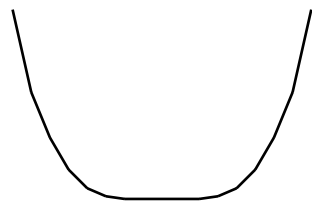
Individuals - objects described by a data set.

Variable - characteristic of an individual.

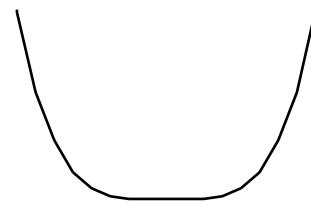
**Example 1.1** Individuals - students, variables - age, gender, GPA, etc.

Observational Studies - observes individuals and measures variables of interest, but does not attempt to influence responses.

Sample Surveys - a type of observational study.



population



sample

**Example 1.2** Public opinion polls, TV ratings, ...

Experiment - deliberately imposes some treatment on individuals to observe their response.

## 1.2 Sampling

Bad ways

(Biased - systematically favor certain outcomes.)

- Convenience sampling - easiest to reach.

- Voluntary response sampling - call-in or write-in polls.

#### Good ways

Simple random samples - SRS of size  $n$  is one chosen in a way such that every set of  $n$  individuals has the same chance of being chosen.

#### Choosing a SRS

1. Assign a numerical label to numbers of the population so that all numbers have the same number of labels.
2. Use a table of random digits (or computer software) to select sample.



population

parameter



sample

statistic

### 1.3 How Sample Surveys Can Go Wrong

#### Sampling Errors

- Random sampling error - variability from sample to sample. Margin of error statements include this only.
- Bad sampling methods - such as convenience sampling.
- Sampling frame / Population issue / Undercoverage (Method: personal interview, phone, mail).

#### Nonsampling Errors

- Mistakes in data entry
- Response errors

- Wording of questions

Questions to ask when reading a poll:

- Who carried out the survey?
- What was the population?
- How was the sample selected?
- How large was the sample? Margin of error?
- What was the response rate?
- How were subjects contacted?
- When was the survey conducted?
- What were the exact questions?

## 1.4 Exercises

Inserted copy of book material.

# 2 Lecture 2

## 2.1 Stratified Random Samples

STEP 1: Divide the sampling frame into distinct groups of individuals, called *strata*. Choose the strata because you have a special interest in these groups within the population or because the individuals within strata are like one another in some ways.

STEP 2: Choose a simple random sample from each stratum.

**Example 2.1** Strata based on either of the following: gender, age group, education level.

This strategy can reduce the margin of error.

## 2.2 Systematic Random Sample

**Example 2.2** Pick a random starting point and sample that and each fifth person (for example). STA 291 may have 130 in a lecture. Pick a number from 1 to 6 at random. Say you pick 3. Now 3, 8, 13, ... form your sample.

## 2.3 Exercises

Inserted book material.

## 2.4 Experiments(Comparative Experimentation)

Response variable - measures outcome of a study.

Explanatory variable - we think it explains or causes changes in the response variable.

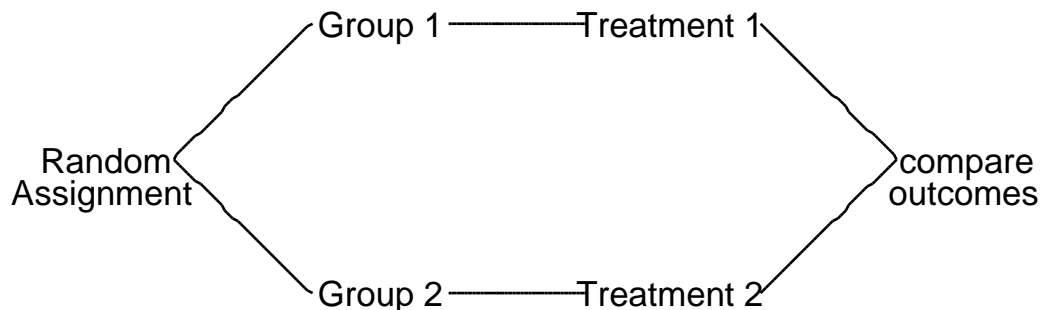
Treatment - specific condition applied to subjects.

**Example 2.3** Online versus classroom course; new drug versus old drug or placebo.

How do we assign individuals to treatments?

- Self-select?
- Selected by experiments, but not necessarily at random?
- Confounding.

Randomized Comparative Experiment



Randomization produces groups that should be similar. Influences other than the treatments operate equally on all groups. Therefore, observed differences in the response variable should be due to the treatments.

So we choose groups like we choose simple random samples.

## 2.5 Exercises

Inserted book material.

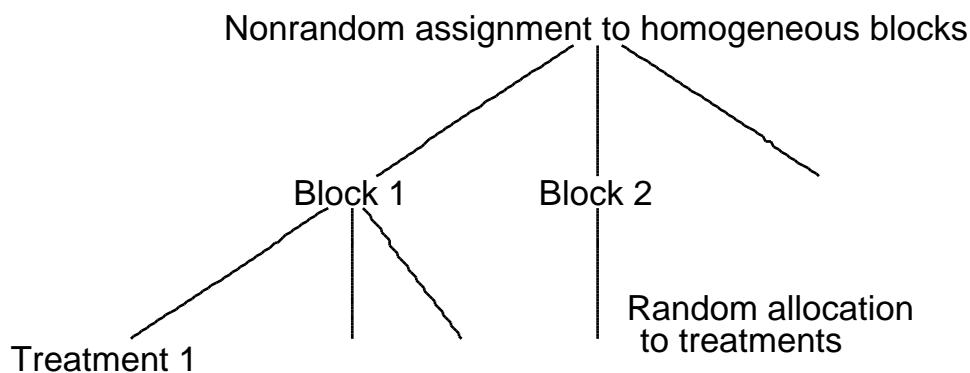
# 3 Lecture 3

## 3.1 Placebo Effect

Double blind experiments - neither the subjects nor the observers know which treatment the subject is receiving. Generalizability.

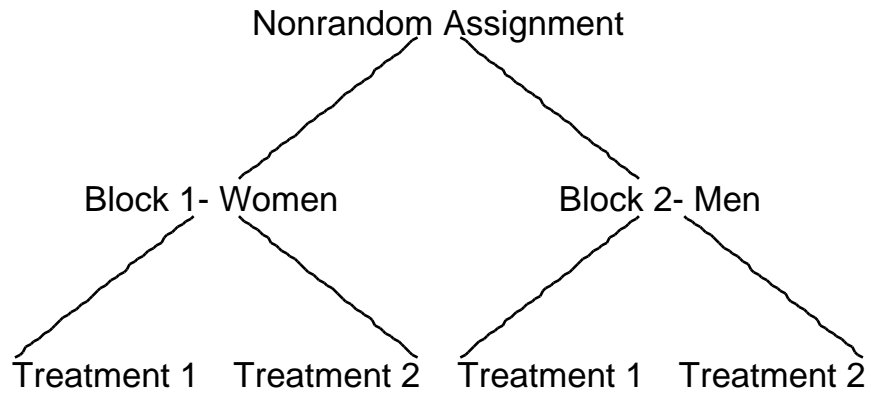
*Another experimental design:*

Randomized Complete Block Design



This is a more elaborate way of controlling for extraneous factors as well as a way which allows one to study effects within blocks. Analogue of stratified random sampling.

**Example 3.1** A clinical trial is investigating a new drug for high blood pressure, Examples of blocks: men/women, over 65/under 65, Caucasian/African-American...



## 4 Lecture 4

### 4.1 Measurement Scales

Nominal - puts things in categories

**Example 4.1** male or female; employed, unemployed, or not in labor force.

Ordinal - puts things in order

**Example 4.2** small, medium, or large; strongly disagree, disagree, neutral, agree, strongly agree.

Interval/Ratio - measures amount of something present.

**Example 4.3** width of room, or temperature.

### 4.2 Organizing and Displaying Data

- Frequency
- Relative frequency
- Histograms
- Stemplots

### 4.3 Examples

Inserted book material.

## 5 Lecture 5

### 5.1 Numerical Summaries/ Graphical Techniques

Challenger data:

84, 49, 61, 40, 83, 67, 45, 66, 70, 69, 80, 58

68, 60, 67, 72, 73, 70, 57, 63, 70, 78, 52, 67

53, 67, 75, 61, 70, 81, 76, 79, 75, 76, 58, 31

- Stemplot
- Measures of Location
- Quartiles
- Five Number Summary
- Boxplot (Box and whisker display)

### 5.2 Measures of location: Mean, Median, Mode

Median - "half above/ half below". Notation:  $\tilde{x}$ .

For  $n = 36$ , there are two "middle" observations, 67 and 68. We take the midpoint of these, 67.5, as the median. If  $n$  is odd, then there is only one "middle" observation and we take that to be the median.

Mean - arithmetic average,  $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i = 65.86$ .

Mode - most frequently occurring value (not necessarily unique). 67 and 70 are modes.



### 5.3 Quartiles and Five number summary

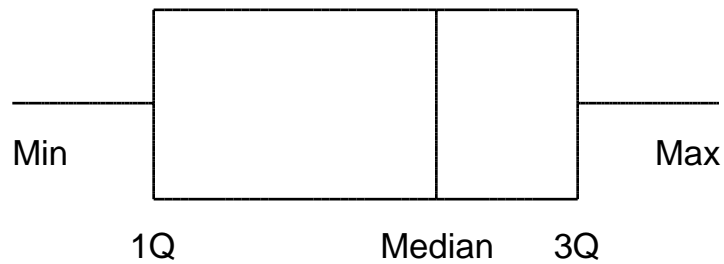
First Quartile - median of observations to the left of the median when ordered from smallest to largest

Third Quartile - median of the observations to the right of the median when ordered from smallest to largest.

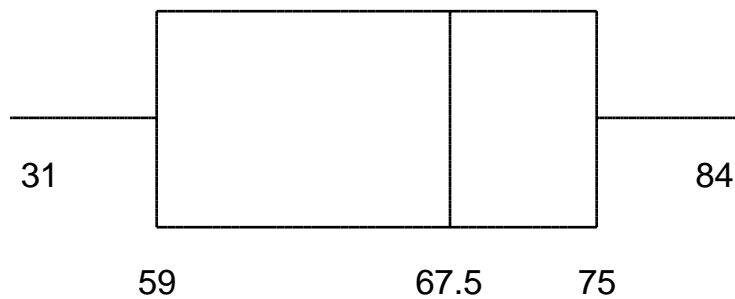
Five number summary

Minimum - First Quartile - Median - Third Quartile - Maximum.

Boxplot.



For the Challenger data: Minimum 31, first quartile 59, median 67.5, third quartile 75, maximum 84.



## 6 Lecture 6

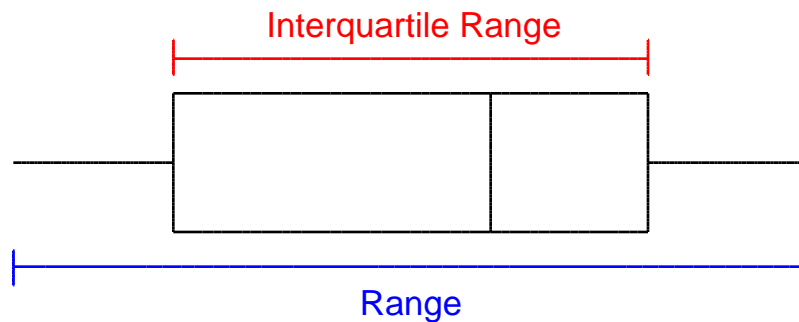
### 6.1 Measures of Spread or Variability

Range = max - min.

Interquartile range = third quartile - first quartile.

Five Number Summary:

minimum - first quartile - median - third quartile - maximum



### 6.2 Variance and Standard Deviation

$$A^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n - 1}$$

Variance is measured in square of original unit.

$$s = \sqrt{(s^2)}$$

Standard deviation is measured in original unit.

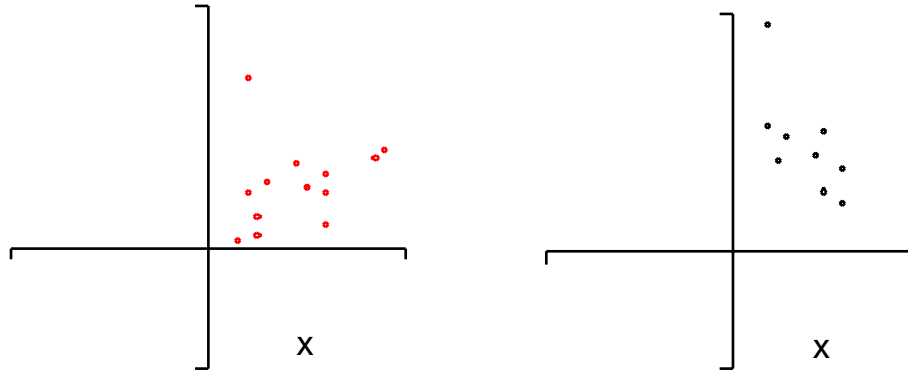
## 7 Lecture 7

### 7.1 Bivariate Data

Examples: (height, weight) or (weight of car, mpg).

*Positive Association* increases in one associated with increases in the other.

*Negative Association* increases in one associated with decreases in the other.



$$(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$$

”Five number summary”

Mean and Standard Deviation of  $x_1, x_2, \dots, x_n$ :

$$\bar{x}, s_x.$$

Mean and Standard Deviation of  $y_1, y_2, \dots, y_n$ :

$$\bar{y}, s_y.$$

*Correlation Coefficient:*

$$r = \frac{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{s_x s_y}$$

Properties of  $r$

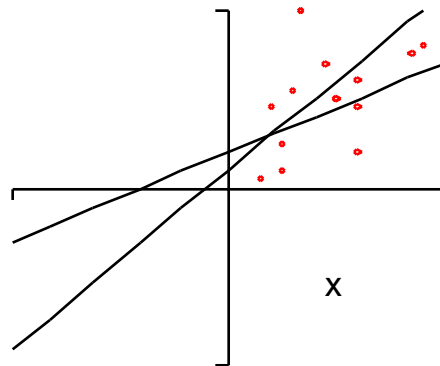
1. measure of association, unitless
2.  $-1 \leq r \leq 1$
3. if  $r \in \{-1, 1\}$ , then points in scatterplot lie on a straight line
4. measures how tightly points cluster about straight line
5. measures straight line association in sense  $r^2$  is the proportion of the variance of one variable that can be explained by straight line relationship.

## 7.2 Examples

Inserted book material (scatterplots with various  $r$ ).

## 7.3 Sample linear Regression

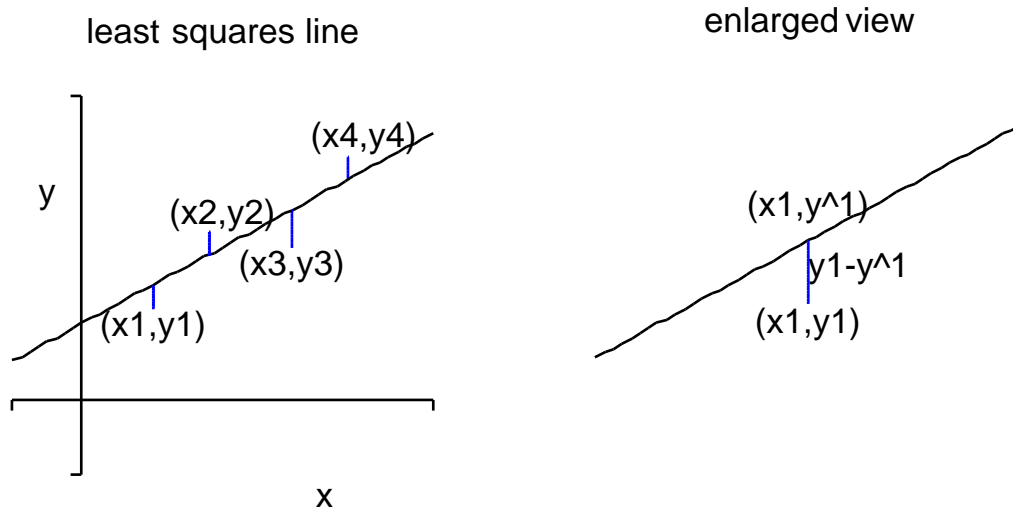
(Regression = Prediction)



Find the line which best fits.

Question: what do we mean by fitting "best"? What criteria should be used?

Least squares line



Given the data  $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ , the least squares line minimizes

$$\sum_{i=1}^n (y_i - \hat{y}_i)^2.$$

**Example 7.1**

$(1, 3), (2, 4), (4, 5), (5, 8)$

Consider  $L_1 : y = x + 2$ . Then  $\hat{y}_1 = 3, \hat{y}_2 = 4, \hat{y}_3 = 6, \hat{y}_4 = 7$ .

$$\sum_{i=1}^4 (y_i - \hat{y}_i)^2 = 0 + 0 + (5 - 6)^2 + (8 - 7)^2 = 2.$$

Consider  $L_2 : y = 2x + 1$ . Then  $\hat{y}_1 = 3, \hat{y}_2 = 5, \hat{y}_3 = 9, \hat{y}_4 = 11$ .

$$\sum_{i=1}^4 (y_i - \hat{y}_i)^2 = 0 + 1 + 16 + 9 = 26.$$

Consider  $L_3 : y = 2x - 1$ . Then  $\hat{y}_1 = 1, \hat{y}_2 = 3, \hat{y}_3 = 7, \hat{y}_4 = 9$ .

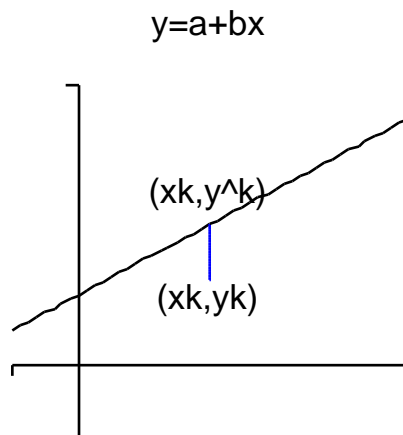
$$\sum_{i=1}^4 (y_i - \hat{y}_i)^2 = 4 + 1 + 4 + 1 = 10.$$

Which of the lines  $L_1, L_2, L_3$  fits best? How do we find the line which best fits? The line which best fits passes through  $(\bar{x}, \bar{y})$  and has slope  $r \frac{s_y}{s_x}$ .

Homework: Find the regression line for the data set of the preceding example.

## 8 Lecture 8

Given the data set  $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ , how can we find the line (i.e. the slope and  $y$ -intercept) which best fits the data in the sense of least squares?



$$\hat{y}_k = \alpha + \beta x_k, k = 1, 2, \dots, n.$$

Find  $\alpha, \beta$  which minimize  $\sum_{k=1}^n (y_k - \hat{y}_k)^2$ . So think of

$$S(\alpha, \beta) = \sum_{i=1}^n (y_i - \alpha - \beta x_i)^2.$$

Then

$$\frac{\partial S}{\partial \alpha} = \sum_{i=1}^n 2(y_i - \alpha - \beta x_i)(-1)$$

$$\frac{\partial S}{\partial \beta} = \sum_{i=1}^n 2(y_i - \alpha - \beta x_i)(-x_i)$$

Set the partial derivatives equal to 0. The resulting equations are equivalent to:

$$\sum_{i=1}^n y_i = n\alpha + \left(\sum_{i=1}^n x_i\right)\beta$$
$$\sum_{i=1}^n x_i y_i = \left(\sum_{i=1}^n x_i\right)\alpha + \left(\sum_{i=1}^n x_i^2\right)\beta$$

Multiply (1) by  $\sum_{i=1}^n x_i$  and (2) by  $n$  and subtract (thereby eliminating  $\alpha$ ):

$$\left(\sum_{i=1}^n x_i\right)\left(\sum_{i=1}^n y_i\right) - n \sum_{i=1}^n x_i y_i = \left[ \left(\sum_{i=1}^n x_i\right)^2 - n \sum_{i=1}^n x_i^2 \right] \beta$$

or, letting  $\hat{\beta}$  represent the value of  $\beta$  solving this:

$$\begin{aligned} \hat{\beta} &= \frac{n \sum_{i=1}^n x_i y_i - \left(\sum_{i=1}^n x_i\right)\left(\sum_{i=1}^n y_i\right)}{n \sum_{i=1}^n x_i^2 - \left(\sum_{i=1}^n x_i\right)^2} \\ &= \frac{\sum_{i=1}^n x_i y_i - \frac{\left(\sum_{i=1}^n x_i\right)\left(\sum_{i=1}^n y_i\right)}{n}}{\sum_{i=1}^n x_i^2 - \frac{\left(\sum_{i=1}^n x_i\right)^2}{n}} \\ &= \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2} = \frac{(n-1)r s_x s_y}{(n-1)s_x^2} = \frac{r s_y}{s_x}. \end{aligned}$$

Substituting  $\hat{\beta} = \frac{r s_y}{s_x}$  into equation (1), we get:

$$\hat{\alpha} = \frac{\sum_{i=1}^n y_i - \left(\sum_{i=1}^n x_i\right)\hat{\beta}}{n} = \bar{y} = \hat{\beta}\bar{x} = \bar{y} - \left(r \frac{s_y}{s_x}\right)\bar{x}.$$

Note that  $(\bar{x}, \bar{y})$  is on the line  $y = \hat{\alpha} + \hat{\beta}x$ .

## 9 Lecture 9

Experiment - A process or phenomenon that can be observed and which has more than one possible outcome.

- Example 9.1**
1. Inspect three items produced along a manufacturing line to see if they conform to specifications.
  2. Roll two dice and observe the outcomes.
  3. Go to the bus stop and wait for the next bus. Observe how long you have to wait.
  4. Observe how many notebooks the University Bookstore will sell in a week.
  5. Observe the amount of bacteria present in a sample of ground beef.

6. Go to the bank and observe how many customers are ahead of you.

Sample space - set of all possible outcomes.

$$S_1 = \{(c, c, c), (c, c, n), (c, n, c), (n, c, c), (c, n, n), (n, c, n), (n, n, c), (n, n, n)\}$$

$$S_2 = \{(1, 1), (1, 2), \dots, (1, 6), (2, 1), \dots, (6, 6)\}$$

$$S_3 = \{x : x \geq 0\} \text{ or } [0, \infty)$$

$$S_4 = \{0, 1, 2, \text{dots}\}$$

The members of the sample space are called *sample points*. An *event* is a collection of sample points.

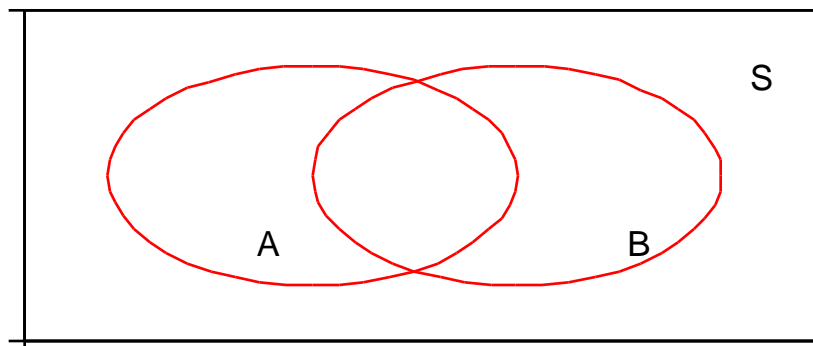
### Example 9.2

$$E_1 = \{(c, c, c), (c, c, n), (c, n, c), (n, c, c)\} = \text{"at most one is nonconforming"}$$

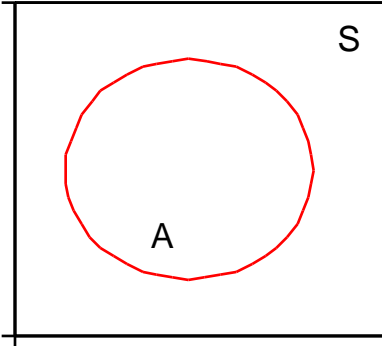
$$E_2 = \{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\} = \text{"sum of faces is 7"}$$

$$E_3 = \{x : x \geq 10\} = \text{wait at least 10 minutes}$$

## 9.1 Venn Diagrams







**Example 9.3** Roll two dice.  $S_1 = \{(1, 1), (1, 2), \dots, (1, 6), (2, 1), \dots, (6, 6)\}$ .

$$A = \{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}$$

$$B = \{(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)\}$$

Then

$$A \cap B = \{(3, 4)\}$$

$$A \cup B = \{(1, 6), (2, 5), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 3), (5, 2), (6, 1)\}$$

De Morgan's Laws:

$$\overline{A \cup B} = \overline{A} \cap \overline{B} \text{ and } \overline{A \cap B} = \overline{A} \cup \overline{B}$$

## 9.2 Assigning probabilities

$S$  = sample space. Let  $A, A_1, \dots$  be events.

Axiom 1 (Rule 1):  $P(A) \geq 0$ .

Axiom 2:  $P(S) = 1$ .

Axiom 3:  $P(A_1 \cup A_2 \cup \dots) = \sum_i P(A_i)$  if the  $A_i$ s are disjoint (have no sample points in common).

Let us illustrate these rules:

$$P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3)$$

$$P(A_1 \cup A_2) \neq P(A_1) + P(A_2)$$

$$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$$

Note also that in general we have  $P(A^C) = 1 - P(A)$ .

Some examples of assigning probabilities:

**Example 9.4** Flip a fair coin twice.

$$S = \{(H, H), (T, H), (H, T), (T, T)\}$$

4 outcomes and they should all have the same probability.  $P(\{(H, H)\}) = \frac{1}{4}$ ,  $P(\{(H, T)\}) = \frac{1}{4}$ , etc. So, what is  $P(\{(H, H), (H, T), (T, H)\})$ ?

**Example 9.5** Suppose the coin is biased in favor of heads. Say that  $\frac{2}{3}$  of the time it will come up with heads. Then:

$$P(\{(H, H)\}) =$$

$$P(\{(H, T)\}) =$$

$$P(\{(T, H)\}) =$$

$$P(\{(T, T)\}) =$$

$$P(\{(H, H), (H, T), (T, H)\}) =$$

Five cards are dealt from a standard 52 card deck. What is the probability that all of the cards are of the same suit?  $S = ?$   $E = ?$

What really is important is how many outcomes there are in the sample space? in the event of interest? (Why? Ans: equally likely outcomes).

We need to have an efficient way to count.

We now establish some counting principles.

Thm 2.1.:  $m$  ways of doing  $a$ ,  $n$  ways of doing  $b$ ,  $mn$  ways of doing  $a$  and  $b$ .

*Question:* Suppose we want to choose a president and a vice-president from a group of five people. In how many ways can we choose?

*Answer:*  $5 \times 4 = 20$  ways.

*Question:* Suppose we choose from a group of  $n$  people.

*Answer:*  $n(n - 1)$  ways.

*Question:* From five people, we choose a president, a vice president, and then a third person who will serve as a secretary/ treasurer. In how many ways can this be done?

*Answer:*  $5 \times 4 \times 3 = 60$  ways.

*Question:* From  $n$  people, we fill  $k$  offices. In how many ways can this be done?

*Answer:*

$$n(n-1)(n-2)\dots(n-(k-1)) = n(n-1)(n-2)\dots(n-k+1) = \frac{n!}{(n-k)!}$$

where  $m! = m(m-1)\dots 3 \times 2 \times 1$  and  $0! = 1$  by definition. [The number of permutations of the  $n$  people taken  $k$  at a time.]

*Question:* From  $n$  people, how many ways are there to choose a committee of  $k$  people?

*Answer:* Let  $\binom{n}{k}$  be the symbol representing the answer. [Called a *binomial coefficient*; recall from algebra that

$$(x+y)^n = \sum_{i=0}^n \binom{n}{i} x^i y^{n-i}.$$

What is the value for  $\binom{n}{k}$ ? Think of it this way:

$$\binom{n}{k} k! = \frac{n!}{(n-k)!} \text{ Why?}$$

or

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

**Example 9.6** A committee of 3 is to be chosen from a group of 10 people, 6 of whom are men and 4 of whom are women. What is the probability that no women are on the committee if everyone has the same chance of being chosen?

How many points are in the sample space? Ans:  $\binom{10}{3}$ .

How many points are in the event? Ans:  $\binom{6}{3}$ .

How much probability does each of these points have? Ans:  $\frac{1}{\binom{10}{3}}$ .

Probability that no women are selected is

$$\frac{\binom{6}{3}}{\binom{10}{3}} = \frac{\frac{6!}{3!3!}}{\frac{10!}{3!7!}} = \frac{7!6!}{3!10!} = \frac{6 \times 5 \times 4}{10 \times 9 \times 8} = \frac{15}{90} = .\overline{16}.$$

**Example 9.7** Suppose that any of the 365 days of the year are equally likely as birthdays. What is the probability that everyone in a room with 23 people, everyone will have a different birthday?

Ans: How many possible birthdays for the 23 people -  $(365)^{23}$ . How many possibilities if all 23 people have different birthdays -  $(365)(364) \dots (343)$ . Probability that all 23 have different birthdays:

$$\frac{(365)(364) \dots (343)}{(365)^{23}} \approx .493.$$

(For 40 people, probability is .109; for 50 people, probability is .03.)

## 10 Lecture 10

### 10.1 Conditional Probability

Roll two fair dice.

$$S = \{(1, 1), (1, 2), \dots, (1, 6), (2, 1), \dots, (6, 6)\}$$

$$A = \{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\} \text{ "roll a sum of 7"}$$

$$B = \{(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)\} \text{ "roll a 3 on die 1"}$$

$$C = \{(4, 6), (5, 5), (6, 4)\} \text{ "roll a sum of 10"}$$

$$D = \{(3, 6), (4, 5), (5, 4), (6, 3)\} \text{ "roll a sum of 9"}$$

$$P(A) = \frac{6}{36}$$

$$P(C) = \frac{3}{36}$$

$$P(D) = \frac{4}{36}$$

What if I know that  $B$  has occurred? Do I want to adjust those probabilities. The conditional probability of  $A$  given that  $B$  has occurred is

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \text{ (provided that } P(B) > 0.)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{36}}{\frac{6}{36}} = \frac{1}{6} = P(A)$$

$$P(C|B) = \frac{P(C \cap B)}{P(B)} = \frac{\frac{0}{36}}{\frac{6}{36}} = 0 \neq P(C)$$

$$P(D|B) = \frac{P(D \cap B)}{P(B)} = \frac{\frac{1}{36}}{\frac{6}{36}} = \frac{1}{6} \neq P(D)$$

Remark: The definition of conditional probability can be rewritten as  $P(A \cap B) = P(B)P(A|B)$  or  $= P(A)P(B|A)$ .

Independence:

$$P(A \cap B) = P(A)P(B).$$

(In other words,  $P(A|B) = P(A)$ .)

In the dice rolling experiment  $A$  and  $B$  are independent.  $C$  and  $B$  are not independent;  $D$  and  $B$  are not independent.

## 10.2 Some other probability rules:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$A \cup B$  can be thought of as  $(A \cap \bar{B}) \cup (A \cap B) \cup (\bar{A} \cap B)$ . Notice these three events are disjoint.

$$\begin{aligned} P(A \cup B) &= P(A \cap \bar{B}) + P(A \cap B) + P(\bar{A} \cap B) = P(A) + P(\bar{A} \cap B) \\ &= P(A) + P(\bar{A} \cap B) + P(A \cap B) - P(A \cap B) = P(A) + P(B) - P(A \cap B). \end{aligned}$$

**Example 10.1**  $A$  = "takes calculus",  $B$  = "takes chemistry". If  $P(A) = .3$ ,  $P(B) = .35$ , and  $P(A \cap B) = .18$ , what is  $P(A \cup B)$ .

Probability of Complements

$$P(\bar{A}) = 1 - P(A) \text{ or } P(A) = 1 - P(\bar{A}).$$

**Example 10.2** A system consists of 3 components only one of which must work for the system to work. Suppose the states (working or failed) of components are independent and each works with probability of .9. What is the probability that the system works?

Solution: The failure probability is  $(.1)(.1)(.1) = .001$  (why?), so the system works with probability .999.

### 10.3 Law of Total Probability

**Example 10.3** Of voters in a certain city, 40% are Republican (R) and 60% are Democrat (D). Among Republicans, 70% favor (F) a bond issue, while 80% of Democrats favor the bond issue. A voter is selected at random. What is the probability that the voter favors the bond issue.

$$\begin{aligned} P(F) &= P(F \cap R) + P(F \cap D) \\ &= P(F|R)P(R) + P(F|D)P(D) = (.7)(.4) + (.8)(.6) = .76. \end{aligned}$$

**Theorem 10.4** Law of Total Probability:(Thm. 2.8.)

$$S = B_1 \cup B_2 \cup \dots \cup B_k$$

$B_i$ 's are disjoint.

$$P(A) = \sum P(A|B_i)P(B_i).$$

Proof:

$$A = (A \cap B_1) \cup (A \cap B_2) \cup \dots \cup (A \cap B_k)$$

$$P(A) = \sum_{i=1}^k P(A|B_i)P(B_i) = \sum_{i=1}^k P(A|B_i)P(B_i)$$

**Example 10.5** A test to detect a particular disease will be positive 80% of the time that the disease is present and negative 95% of the time when the disease is absent. Suppose that 10% of those screened have the disease. What percent will have a positive test?

+ = have positive test  $D$  = have the disease.

$$\begin{aligned}
P(+) &= P(+ \cap D) + P(+ \cap D^c) \\
&= P(+|D)P(D) + P(+|D^c)P(D^c) = (.98)(.10) + (.05)(.90) = .098 + .045 = .143.
\end{aligned}$$

14.3% will have a positive test.

Returning to second example:  
 Suppose a person has a positive test. What is the probability that the person has the disease?

$$\begin{aligned}
P(D|+) &= \frac{P(D \cap +)}{P(+)} = \frac{P(+|D)P(D)}{P(+)} = \frac{P(+|D)P(D)}{P(+|D)P(D) + P(+|D^c)P(D^c)} \\
&= \frac{(.98)(.10)}{.143} \approx .685.
\end{aligned}$$

**Theorem 10.6** Bayes Rule: (Thm. 2.9.)

$$S = B_1 \cup B_2 \cup \dots \cup B_k$$

$B_i$ 's are disjoint.

$$P(B_j|A) = \frac{P(A|B_j)P(B_j)}{\sum_i P(A|B_i)P(B_i)}.$$

Years ago, a television show called "Let's Make a Deal" was popular. It was hosted by Monty Hall. The idea was essentially this: The contestant is shown three doors. Behind one of them is a valuable prize. Behind the other two doors are booby prizes. After the contestant chooses a door, Monty then shows him/her what is behind a different door. It is always a booby prize. (Since there are two doors with booby prizes, Monty always has at least one of the unchosen doors that can be shown to have a booby prize. Let us assume that when the contestant has selected the door with the valuable price, Monty picks one of the two remaining doors at random to show the contestant). now the contestant is offered a chance to change his/her mind and take the other door.

For example,

1. Contestant chooses door #1
2. Monty shows door #3 and it is a booby prize
3. Monty now tells contestant, that he or she can stay with door #1 or switch to #2.

Is it to the contestant's advantage to stay or switch? Or are they equally advantageous. The question arose in Marilyn Vos Savant's column years ago and is in her new book "The Power of Logical Thinking" (which I think you would enjoy). Marilyn gets it right. Many of her readers do not. Her explanation does not involve formal probability, but I claim it is a simple application of Bayes Rule. See if you can answer the question now that you know Bayes Rule.

(You might wish to play around with this by simulation.

Go to my home page [www.ms.uky.edu/~griffith](http://www.ms.uky.edu/~griffith), click on the link to STA/MA320 course page, choose the link for the Monty Hall problem.)