## 14. Circles On The Plane

***Problem* 14.1.** (HA) Find the center $C$ of the circle passing through points $P = (14, 12)$, $Q = (-11, 7)$ and $R = (22, -2)$.

**Hint(s) to 14.1**: $C$ is equidistant to all three points.

**Answer to 14.1**: $(4, -3)$

***Problem* 14.2.** Show that each circle on the plane has equation of the form $z \cdot \bar{z} + a \cdot z + b \cdot \bar{z} + c = 0$, where $a$ is the conjugate of $b$, $b \neq 0$, $c$ is real, and $|a|^2 > c$.

***Problem* 14.3.** Suppose $z\cdot\bar{z}+a\cdot z+b\cdot\bar{z}+c=0$ is an equation of a circle on the plane. Prove that $a$ is the conjugate of $b$, $b\neq 0$, $c$ is real, and $|a|^2>c$.

***Problem* 14.4.** (A) The circle centered at $1+2\cdot i$ of radius 3 has equation $z\cdot\bar{z}+a\cdot z+b\cdot\bar{z}+c=0$, where $c$ is real. Find $c$.

**Answer to 14.4**: $-4$

***Problem* 14.5.** Show that a circle and a line intersect at at most 2 points.

***Problem* 14.6.** Let $f(z) = z + a$, where $a$ is a constant. Show that if $C$ is a circle, then $f(C)$ is a circle, too.

***Problem* 14.7.** Let $f(z) = \bar{z}$. Show that if $C$ is a circle, then $f(C)$ is a circle, too.

***Problem* 14.8.** Let $f(z) = 1/z$. Show that if $L$ is a line not passing through 0, then $f(L)$ is a circle.

***Problem* 14.9.** Let $f(z) = 1/z$. Show that if $C$ is a circle not passing through 0, then $f(C)$ is a circle, too.

***Problem* 14.10.** Let $f(z) = 1/z$. Show that if $C$ is a circle passing through 0, then $f(C)$ is a line.

***Problem* 14.11.** Let $f(z) = a \cdot z + b$, where $a$ and $b$ are constant. Show that if $C$ is a circle, then $f(C)$ is a circle, too.

***Problem* 14.12.** Let $f(z) = (a \cdot z + b)/(c \cdot z + d)$, where $a$, $b$, $c$, and $d$ are constant so that $ad - bc \neq 0$. Show that if $X$ is a line or a circle, then $f(X)$ is a circle or a line.

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