DAY ONE

Domains of Mathematical Knowledge for Teaching

1. Common Content Knowledge. "[T]he mathematical knowledge and skill used in settings other than teaching." Example: Knowing the algorithm to multiply together two numbers.

2. Specialized Content Knowledge. "[T]he mathematical knowledge and skill unique to teaching." Example: Knowing how the algorithm to multiply together two numbers connects to place value and the distributive property.

3. Horizon Content Knowledge. "[A]n awareness of how mathematical topics are related over the span of mathematics included in the curriculum." Example: Knowing how the algorithm to multiply together two numbers is related to multiplying together two polynomials.

4. Knowledge of Content and Students. "[K]nowledge that combines knowing about students and knowing about mathematics. Teachers must anticipate what students are likely to think and what they will find confusing." Example: Knowing that when multiplying together two numbers students may make the error of not appropriately "shifting" the terms to be added.

5. Knowledge of Content and Teaching. "[C]ombines knowing about teaching and knowing about mathematics. Many of the mathematical tasks of teaching require a mathematical knowledge of the design of instruction." Example: Knowing what teaching strategies to employ so that students, when multiplying two numbers, learn how and why to appropriately "shift" the terms to be added.

6. Knowledge of Content and Curriculum. "[R]epresented by the full range of programs designed for the teaching of particular subjects and topics at a given level, the variety of instructional materials available in relation to these programs, and the set of characteristics that serve as both the indications and contraindications for the use of particular curriculum or program materials in particular circumstances." Example: Knowing what instructional materials are available for teaching and learning multiplication of two numbers, what approach these materials take, and how effective they are.

Some Major Points — Patterns

1. There is always more than one way to extend the first terms of a pattern.

2. Patterns can be described by recursive (iterative) formulas and explicit (closed) formulas. Sometimes one is easier to recognize than the other.

3. Formulas can be recognized and justified by a variety of methods; e.g., algebraically, geometrically, using finite differences, using curve-fitting.

4. There is a difference between recognizing or identifying a formula or pattern, and proving that it is true for all values.

5. "In the middle grades, the use of techniques such as finding common differences and applying procedures by rote to determine function rules on the basis of common differences is not appropriate unless students can justify their reasoning through evidence from, and reference to, the models of the pattern being considered." (*Navigating Through Algebra Grades 6–8*, p. 17)

Guess My Pattern

Consider a sequence that begins:

1, 2, 3, _____, _____, _____, . . .

Each group should think of a rule or formula that fits this sequence and write in the next three terms. Your goal is to come up with a pattern that you don't think any other group will think of!

Record your rule here:

On a single poster, all groups should write their next three terms (but not the rules!). Then everyone should try to guess the various rules.

Guess My Pattern, Continued

Some questions for subsequent discussion:

1. Does this activity help break the misconception that there is only one way to continue a given sequence?

2. Of the various rules produced by the groups, which ones were explicit and which were recursive?

3. What are some ways to guess the rules?

4. Which rules will students find easier or harder to guess? Why?

5. What strategies can improve students' rule-guessing skills?

Building with Toothpicks

1. Carry out the activity "Building with Toothpicks" from *Navigating through Algebra Grades 6–8*. Each group should record its response on a poster.

2. Examine the results from each group as well as the three samples of student work.

   (a) How did each group or student approach the problem?

   (b) Did any approach seem to involve developing an explicit formula?

   (c) Did any approach seem to involve developing a recursive formula?

   (d) To what extent did groups or students justify their patterns or formulas?

   (e) What could the teacher do help students develop more complete justifications?

Odd Addition

Consider the problem of finding the sum of the first n positive odd integers.

1. Make a table of values:

n (number of terms)	$s(n)$ (sum)
1	1
2	$1 + 3 = ?$
3	$1 + 3 + 5 = ?$
4	
5	
6	
7	
8	
9	
10	

2. What pattern do you detect? How can it be justified? Record your responses on a poster.

Odd Addition, Continued

Some questions for subsequent discussion:

1. What is the explicit formula? Try to justify it using an $n \times n$ grid.

2. What is the recursive formula? Show algebraically that the explicit formula satisfies the recursive formula.

3. What is the difference between detecting a formula and justifying a formula?

4. What teaching strategies can assist students in recognizing the difference between formula detection and formula justification?

What is the Pattern?

1. Draw a circle and mark $n \geq 1$ points around the circumference. Connect these points by line segments (chords of the circle) in every possible way, adjusting the locations of the points, if necessary, so that not more than two segments cross at any inside point. Work out some examples and make a conjecture as to how many regions the circle will be divided into if $n = 1, 2, 3, 4, 5, 6, \ldots$? (If $n = 1$ you cannot draw any segments, so there is only one region.)

2. Additional problems.

   (a) Formulate a rule or formula for each of the following sequences:
      i. $1, 2, 3, 4, \ldots$.
      ii. $1, 2, 3, 5, \ldots$.
      iii. $1, 2, 3, 6, \ldots$.
      iv. $1, 2, 3, c, \ldots$ for arbitrary c.

   (b) Create your own examples of sequences that can be continued in different ways using different rules or formulas.

What is the Formula?

1. What is the sum of the first n positive odd integers?

2. Use an $n \times n$ grid of squares to prove your formula.

3. What is the sum of the first n positive integers? First do some experiments to guess the formula, and then find some way to prove it. You may wish to consider algebra, or perhaps some geometric figure, or by graphing.

4. What is a formula for the sum $1 + 2 + 4 + 8 + 16 + \cdots + 2^n$? First do some experiments to guess the formula, and then find some way to prove it.

Explicit vs. Recursive

1. Consider the following pattern:

n	1	2	3	4	5
$f(n)$	2	5	8	11	14

   (a) An *explicit formula* (or *closed formula*) expresses $f(n)$ directly in terms of n. What is an explicit formula that works for this pattern?

   (b) A *recursive* or *iterative* formula expresses $f(n)$ in terms of one or more preceding terms, like $f(n-1)$, and also tells you what the first term is, like $f(1)$, so you know how to get started. What is a recursive formula that works for this pattern?

2. We can sometimes detect a recursive formula by taking *differences* of our values. For example, what recursive formula can we detect from the following pattern?

n	1	2	3	4	5
$f(n)$	2	8	17	29	44
	6	9	12	15	

   Answer: $f(1) = 2$ and thereafter $f(n) = f(n-1) + 3n$.

3. Sometimes the explicit formula is easier to detect than the recursive formula. For example:

n	1	2	3	4	5
$f(n)$	1	8	27	64	125

   What is an explicit formula? Can you think of another example where the explicit formula is easier to detect?

   Sometimes the recursive formula is easier to detect than the explicit formula. For example:

n	1	2	3	4	5	6
$f(n)$	1	2	3	5	8	13

   What is a recursive formula? Can you think of another example where the recursive formula is easier to detect?

Explicit and Recursive in Harmony

If you have a guess for both the explicit formula and the recursive formula, and you know that one of them is true, then you can check the other one by verifying that the two formulas are compatible. You can do this by checking the initial value is the same for both, and that you get a true statement when you substitute the explicit formula into the recursive formula.

For example, if you compute the sum of the numbers $1, \ldots, n$, you may *suspect* that the formula is $f(n) = \frac{n(n+1)}{2}$ because that formula works for low values, but you might not have a proof.

n	1	2	3	4	5	6
$f(n)$	1	3	6	10	15	21

But you *know* that $f(1) = 1$ and thereafter $f(n) = f(n-1) + n$. So you can *prove* $f(n) = \frac{n(n+1)}{2}$ by checking that $f(1)$ yields 1 (yes!), and then checking that the recursive formula is satisfied:

$$f(n) = f(n-1) + n?$$

$$\frac{n(n+1)}{2} = \frac{(n-1)(n-1+1)}{2} + n?$$

$$\frac{n(n+1)}{2} = \frac{n^2 - n}{2} + \frac{2n}{2}?$$

$$\frac{n(n+1)}{2} = \frac{n^2 + n}{2}, \text{ True!}$$

1. Prove that the sum of the first n positive odd integers is equal to n^2 by (1) explaining why the recursive formula below is true, and (2) verifying that the recursive formula and the proposed explicit formula are compatible:

   - Recursive Formula: $f(1) = 1$ and thereafter $f(n) = f(n-1) + (2n-1)$.
   - Explicit Formula: $f(n) = n^2$.

2. Prove that the sum of squares of the first n positive integers is given by

   $$f(n) = \frac{1}{6}n(n+1)(2n+1)$$

   by showing that it is compatible with the recursive formula $f(1) = 1$ and thereafter $f(n) = f(n-1) + n^2$.

Deriving Formulas from Finite Differences

Even though we should be cautious about the blind application of finite differences to detect formulas without subsequent justification and understanding, nevertheless it is an extremely useful method for guessing formulas for patterns given by polynomial functions. See, for example, Chapter 14, "Evaluate Finite Differences," in the book *Crossing the River with Dogs,* by Johnson, Herr, and Kysh.

DAY TWO

Some Major Points — Functions

1. "Explorations that develop from problems that can be solved by using tables, graphs, verbal descriptions, concrete or pictorial representations, or algebraic symbols offer opportunities for students to build their understanding of mathematical functions." (*Navigating Through Algebra Grades 6–8*, p. v)

2. There are differences between the graph of the position of an object as a function of time and the actual path traced by the object.

3. Graphs of distance, velocity, and acceleration as functions of time are related to each other. "Graphs of distance versus time and speed versus time pose different kinds of challenges. It is important to help students distinguish between the two situations. Most students confuse distance with speed. Students need opportunities to make and to read both kinds of graphs in which speed remains constant for different intervals of time." (*Navigating Through Algebra Grades 6–8*, p. 31)

4. Different variables can often be used as either independent or dependent variables.

5. Functions may sometimes be described with additional parameters.

From Stories to Graphs

1. Carry out the activity "From Stories to Graphs" from *Navigating through Algebra Grades 6–8*. Each group should record its response on a poster.

2. Do students have difficulty sketching a graph when just a few or no numbers are specified?

3. Do students have difficulty deciding which variable to choose to be independent and which to be dependent?

4. Do students have difficulty distinguishing between the path of an object that is moving (as in Josephine's walk) and the graph that results?

5. Make up your own story for your students to graph. Why do you think it is a good story for gaining insights into potential student misconceptions?

From Graphs to Stories

1. Carry out the activity "From Graphs to Stories" from *Navigating through Algebra Grades 6–8*. Each group should record its responses on a poster.

2. What difficulties do you anticipate your students to have with these problems? How might you respond?

3. Examine the graph in Figure 2.12, *Navigating Through Algebra Grades 6–8*, p. 31. Why is this an impossible graph?

4. Examine the graphs in Figure 2.13, *Navigating Through Algebra Grades 6–8*, p. 31. Are either of them possible? Impossible?

5. Examine the graph in Figure 2.14, *Navigating Through Algebra Grades 6–8*, p. 32. Write a story about Tony's walk. Each group should record its response on a poster.

6. Examine the groups' responses as well as the three student responses. How did each group or student approach the problem?

7. Make a graph of Tony's distance walked as a function of time.

8. Make up your own speed versus time graph for your students to describe in a story. Why do you think it is a good graph for gaining insights into potential student misconceptions?

On Being the Right Size

A large balloon animal (such as those used in parades) is being inflated from a small size to a larger size with a scaling factor of k.

1. Sketch the shape of the graph of the length of the balloon as a function of k.

2. Sketch the shape of the graph of the surface area of the balloon as a function of k.

3. Sketch the shape of the graph of the volume of the balloon as a function of k.

4. Sketch the shape of the graph of the volume of the balloon as a function of the surface area.

Throwing Rocks

1. If you throw a rock straight up into the air with an initial height of 12 feet and an initial velocity of 16 feet/sec, its height as a function of time is given by $h(t) = -16t^2 + 16t + 12$ feet. Graph the height as a function of time. What kind of a curve results?

2. Provide physical interpretations for the intercepts and the vertex.

3. Sketch and discuss the velocity of the rock as a function of time.

4. How far away from where you are standing does the rock hit the ground?

A Nice Fall

1. If you drop a rock off a bridge, then the distance it has fallen at the end of t seconds is given by $s(t) = 16t^2$ feet. Graph the distance as a function of time.

2. The rock's velocity at time t is given by $v(t) = 32t$ feet/sec. Graph the velocity as a function of time.

3. The rock's acceleration at the end of t seconds is given by $a(t) = 32$ feet/sec^2. Graph the acceleration as a function of time.

4. How do the above three graphs relate to each other?

5. Graph the velocity of the rock as a function of the distance fallen. What kind of curve is this?

6. Suppose you estimate the height of a bridge over a river by dropping a rock and listening for the sound of the splash. You hear the splash exactly 2 seconds later. Knowing that the speed of sound in air is 1125 ft/sec, how high is the bridge?

Nice Rectangles

A rectangle has dimensions $x \times y$.

1. Express its area A as a function of x and its perimeter P.

2. If the perimeter P equals 10, what dimensions yield the rectangle with the largest area?

3. On the other hand, express the perimeter P of an $x \times y$ rectangle as a function of its area A.

4. If the area A equals 10, what dimensions yield the rectangle with the smallest perimeter?

DAY THREE

Some Major Points — Probability

1. Students need to learn to "understand and apply basic concepts of probability." (*Navigating Through Probability Grades 6–8*, p. 2)

2. Students should develop and refine their understanding of chance; become aware of the notions of complementary events and mutually exclusive events; investigate compound events and use tree diagrams, organized lists, or similar descriptive methods to determine probabilities in such situations; have ample experience with hands-on experiments and computer simulations that produce large data sets. (*Navigating Through Probability Grades 6–8*, pp. 7–8)

3. The formulas for permutations and combinations can be derived using the multiplication counting principle.

4. It is important to correctly identify the set of possible outcomes and their probabilities.

5. It is important to understand the notion of independence.

6. Expected value is just one piece of information you may need when making decisions.

7. Understanding conditional probability.

Dice Differences

1. Carry out the activity "Dice Differences ," from *Navigating through Probability Grades 6–8*. Each group should record its response on a poster.

2. In what ways will this activity:

   (a) Help students understand sample space and associated numeric outcomes?

   (b) Help students learn to organize the results of a random experiment?

   (c) Help students understand the pooling of results?

   (d) Help students calculate experimental probabilities?

3. What is the theoretical probability that player A will win? That player B will win?

4. How could you change the rules to make the game more fair?

First Head

1. Carry out the activity "First Head," Exploration 1, from *Navigating through Probability Grades 6–8*. Each group should record its response on a poster.

2. Calculate the corresponding theoretical probabilities and compare them with the experimental probabilities.

3. How can a tree diagram help calculate these probabilities? How can the multiplication principle help calculate these probabilities?

4. What difficulties are students likely to have with this activity? How might you respond?

Order, Order!

1. You have four different books in a box. How many different ways can you place these four books on a shelf?

2. There are 10 books scrambled in a box: Algebra, Calculus, Discrete Mathematics, Geometry, Number Theory, Pre-Calculus, Probability, Problem Solving, Statistics, and Trigonometry. You randomly grab four books and line them up on your bookshelf. What is the probability that you end up with the following four books sitting on your shelf in this order: Algebra, Trigonometry, Pre-Calculus, and Calculus?

3. Same as above, but this time you randomly grab four books and stuff them into your backpack. What is the probability that you get Algebra, Trigonometry, Pre-Calculus, and Calculus?

4. What is the general formula for the solution to problem (2) if you have n books in the box and you randomly grab m of them? What does this formula make sense—how can you derive it from scratch and justify it?

5. What is the general formula for the solution to problem (3) if you have n books in the box and you randomly grab m of them? What does this formula make sense—how can you derive from scratch and justify it?

The Second Girl

1. Suppose that a woman is equally likely to give birth to a boy or a girl (this is not exactly true, though). If she has two children and one of them is a girl, what is the probability that the other child is a girl? James thinks the answer is 1/2. How can you address his error in thinking? What misconception has tripped him up?

2. Suppose that a woman has two children, and the first child is a girl? What is the probability that the second child is a girl?

3. Suppose that a woman has three children and at least one of them is a girl. What is the probability that at least one of the other two children is also a girl?

4. What are some other activities that can assist students to understand the notion of possible outcomes and conditional probability?

What Should We Expect Next?

1. You have just flipped a fair coin ten times and achieved the sequence $HHHHHHHHHH$. Sarah thinks that with so many heads occurring already, that tails is long overdue and there is a fairly high probability that the next toss will be tails. What is her misconception and how can you address it? What is the probability that the next toss is going to result in T?

2. Flip a coin ten times and record your results:

   _____ _____ _____ _____ _____ _____ _____ _____ _____ _____

   What is the probability of flipping a fair coin ten times and achieving the sequence that you just achieved? Will students have difficulty with this answer?

3. What is the probability of flipping a fair coin ten times and achieving the sequence $HHHHHHHHHH$?

4. What is the probability of flipping a fair coin ten times and achieving the sequence $HHTHTTTHHH$? Will students be troubled when comparing the answer to this problem with that of the previous problem?

5. Should you view the outcome of (3) with more suspicion than (4)? Why or why not?

6. What is the probability of flipping a fair coin ten times and achieving ten heads?

7. What is the probability of flipping a fair coin ten times and achieving six heads?

8. Suppose your coin is not fair, and the probability of getting a head on a flip is p. What value of p maximizes the probability of getting six heads?

Strange Dice

Suppose you have three dice. The sides of die A are labeled 1, 1, 6, 6, 8, 8. The sides of die B are labeled 3, 3, 5, 5, 7, 7. The sides of die C are labeled 2, 2, 4, 4, 9, 9.

1. If you make one roll of dice A and B, what is the probability that A will beat B?

2. If you make one roll of dice B and C, what is the probability that B will beat C?

3. If you make one roll of dice A and C, what is the probability that A will beat C?

4. Which die is the "best"? Are you surprised?

The Gambler's Scheme

A gambler claims to have a fool-proof scheme to make some money gambling. He is at a (fictitious) casino at which there is a coin-flipping game. The player places a bet of x dollars and the fair coin is flipped. If the result is heads, the player gains x dollars, but if the result is tails, the player loses x dollars. Here is the proposed scheme:

1. Bet $1. If you win, you have gained a dollar. But if you lose, then:

2. Bet $2. If you win, you have lost $1 but gained $2, so you have a net gain of $1. But if you lose, then:

3. Bet $4. If you win, you have lost $1 plus $2 but gained $4, so you have a net gain of $1. But if you lose, then, etc.

In other words, double your your bet after every loss, and then as soon as you win, you will have gained a dollar. So you are guaranteed to come out ahead by $1!

Does this scheme work? Why or why not?

It's Your Birthday!

Ignore seasonal fluctuations and the presence of leap years and assume that any particular person's birthday is equally likely to be any of the 365 days of the year. What is the probability that in a room of 25 random people there are at least two with the same birthday?

Expectations Aren't Everything

1. Suppose you are going to play a game in which you may either win or lose money with certain probabilities, but you know the expected value of the game is \$10. Would you play?

2. Suppose you are going to play a game in which you may either win money with probability $\frac{1}{2}$ or lose money with probability $\frac{1}{2}$. If you know the expected value of the game is \$10, would you play?

3. Suppose you are going to play a game in which you may either win \$30 with probability $\frac{1}{2}$ or lose \$10 with probability $\frac{1}{2}$. What is the expected value of the game? Would you play?

4. Suppose you are going to play a game in which you may either win \$5000030 with probability $\frac{1}{2}$ or lose (be forced to pay) \$5000010 with probability $\frac{1}{2}$. What is the expected value of the game? Would you play?

Ill or Not?

Suppose it is known that 0.2% of the people in your town have a particular disease (or, perhaps, a certain gene). There is a test for this disease that is correct 99% of the time—that is to say, if someone has the disease, then the test correctly reports that they do 99% of the time; and if they do not have the disease, then the test correctly reports that they do not 99% of the time. Suppose the disease is spread at random throughout the people in your town (though this is typically not the case) and you test positive for the disease. What is the probability that you actually have the disease? Make a guess first before working out the answer.

TECHNOLOGY

Applets in NCTM Books

1. *Navigating Through Algebra Grades 6–8*

   (a) Recursive grapher

   (b) Simple plot

2. *Navigating Through Probability Grades 6–8*

   (a) Spinner

   (b) Adjustable spinner

   (c) Racing game with two dice

National Library of Virtual Manipulatives
`http://nlvm.usu.edu`

A large collection of applets sorted according to content strand and grade level. Here is a sample.

1. Algebra

    (a) Algebra Tiles

    (b) Function Machine

    (c) Grapher

    (d) Towers of Hanoi

2. Data Analysis and Probability

    (a) Coin Tossing

    (b) Hamlet Happens

    (c) Pascal's Triangle

    (d) Scatterplot

    (e) Spinners

    (f) Stick or Switch

An online site for a huge range of computational possibilities. Here are some examples of commands to try.

1. Factoring:
   `factor x^8-1`

2. Expanding:
   `expand (x+1)(x+2)(x+3)`

3. Solving:
   `solve x^5-x^4-5x^3+7x^2-6x+8=0`
   `solve |x-5|>7`
   `solve |x-5|<=7`

4. Plotting:
   `plot x^5-x^4-5x^3+7x^2-6x+8`
   `plot x^2-y^2, x=-1 to 1, y=-1 to 1`
   `plot {x^2,x^3}`

5. Finding functions of best fit:
   `linear fit {{1,1}, {2,2}, {3,3}, {4,6}}`
   `quadratic fit {{1,1}, {2,2}, {3,3}, {4,6}}`
   `cubic fit {{1,1}, {2,2}, {3,3}, {4,6}}`
   `quartic fit {{1,1}, {2,2}, {3,3}, {4,6}}`
   `quintic fit {{1,1}, {2,2}, {3,3}, {4,6}}`

6. Sequences:
   `1,4,9,16,25,...`
   `1/2+1/4+1/8+1/16+...`

7. Sums and Products:
   `sum k^3, k=1 to 10`
   `sum k^3, k=1 to n`
   `sum (1/3)^k, k=0 to infinity`
   `g(n)=n^2+g(n-1), g(0)=0`
   `365/365 * 364/365 * 363/365 * ... * 341/365`

8. Optimization:
```
maximize 5x-x^2
minimize 20/x+2x, x>0
```

9. Counting:
```
10 choose 3
n choose m
10 permutation 3
```

10. Probability:
```
32 coin tosses
probability 12 heads 8 tails
5 dice
two 12-sided dice
probability 3 queens 2 jacks
probability full house
Kentucky lottery
```

11. Geometry:
```
sphere
torus
dodecahedron
```

12. Calculus:
```
derivative of x^3+x^2
integral of x^3+x^2
integral of x^3+x^2 from x=1 to x=2
```

13. Chemistry:
```
cyclohexane
```

14. Geography:
```
Paris, Kentucky
```

15. Astronomy:
```
Mars
Big Dipper
```

Using Spreadsheets to Implement Explicit and Recursive Formulas

1. Example: The explicit formula $f(n) = 2^n - 1$, $n = 1, 2, 3, \ldots$.

   (a) Type 1 in cell A1 and 2 in cell A2.

   (b) Click and drag to highlight both cells A1 and A2.

   (c) Click and drag down the tiny square in the lower right-hand corner of the highlighted area, dragging down about 20 cells. The result should be a column of consecutive whole numbers.

   (d) Enter =2^A1-1 in cell B1.

   (e) Click cell B1 to highlight it.

   (f) Click and drag down the tiny square in the lower right-hand corner of the highlighted area. The result should be a column of the values of $f(n)$.

   (g) Click various cells in the second column and confirm at the top of the spreadsheet that the correct formula is being used in each case. Notice how each cell Bi in the second column is computed using the value of n in cell Ai of the first column. (You can also hold down the control key and type ', the single left quotation mark, to toggle back and forth from viewing the values and viewing the formulas in the cells.)

2. Example: The recursive formula $f(1) = 1$ and thereafter $f(n) = 2f(n - 1) + 1$, $n = 2, 3, 4, \ldots$.

   (a) Type 1 in cell A1 and 2 in cell A2.

   (b) Click and drag to highlight both cells A1 and A2.

   (c) Click and drag down the tiny square in the lower right-hand corner of the highlighted area, dragging down about 20 cells. The result should be a column of consecutive whole numbers.

   (d) Enter 3 in cell B1.

   (e) Enter =2*B1+1 in cell B2.

   (f) Click cell B2 to highlight it.

   (g) Click and drag down the tiny square in the lower right-hand corner of the highlighted area. The result should be a column of the values of $f(n)$.

   (h) Click various cells in the second column and confirm at the top of the spreadsheet that the correct formula is being used in each case. Notice how each cell Bi in the second column is computed using the value in cell B$i - 1$ of the second column.

Using Spreadsheets to Make Tables of Differences

Example: The sequence $f(n) = 3, 7, 13, 21, 31, 43, \ldots$ for $n = 1, 2, 3, \ldots$.

1. Type `1` in cell `A1` and `2` in cell `A2`.

2. Click and drag to highlight both cells `A1` and `A2`.

3. Click and drag down the tiny square in the lower right-hand corner of the highlighted area, dragging down about 20 cells. The result should be a column of consecutive whole numbers.

4. Enter the terms of the sequence $3, 7, 13, 21, 31, 43$ in the second column, in cells `B1` through `B6`.

5. Enter `=B2-B1` in cell `C1`.

6. Click cell `C1` to highlight it.

7. Click and drag down the tiny square in the lower right-hand corner of the highlighted area, through cell `C5`. The result should be a column of differences.

8. Click various cells in the third column and confirm at the top of the spreadsheet that the correct difference is being computed in each case.

Using Spreadsheets to Make Scatterplots and Approximate with Curves

Example: Let's consider the sequence

n	1	2	3	4	5	6
$s(n)$	3	7	13	21	31	43

We will plot this sequence as points $(1,3)$, $(2,7)$, $(3,13)$, $(4,21)$, $(5,31)$, $(6,43)$. Note: Commands may be different on different spreadsheets, so you may have to explore your options.

1. Enter the first coordinates in column A: $1, 2, 3, 4, 5, 6,$.

2. Enter the second coordinates in column B: $3, 7, 13, 21, 31, 43$.

3. Highlight columns A and B by holding down the control key and clicking on A and then on B at the top of the columns.

4. Click Insert, then click Charts, then click Scatter, then click on the scatterplot diagram.

5. Click on the chart that is created. Then click Layout, click Analysis, click Trendline, and finally click Linear Trendline. Or finish up by clicking More Trendline Options..., selecting Polynomial with Order, say, 3, clicking Display Equation on chart, and finally clicking Close.

Using Graphing Calculators to Make Scatterplots and Approximate with Curves

Example: Let's consider the sequence

n	1	2	3	4	5	6
$s(n)$	3	7	13	21	31	43

We will plot this sequence as points $(1,3)$, $(2,7)$, $(3,13)$, $(4,21)$, $(5,31)$, $(6,43)$. Note: Commands may be different on different calculators, so you may have to explore your options. The instructions below are for the TI-73 Explorer.

1. Press 2nd Mem, press 6 to select ClrAllLists, and press ENTER to clear all lists.

2. Press LIST to see your lists. Enter the first coordinates, $1, 2, 3, 4, 5, 6$, in list L1, and the second coordinates, $3, 7, 13, 21, 31, 43$, in list L2.

3. Press 2nd PLOT to enter STAT PLOTS.

4. Place the cursor on 1: and press ENTER.

5. On the line Plot1 place the cursor on ON and press ENTER.

6. On the next line place the cursor on the first symbol (for scatterplot) and press ENTER.

7. On the line Xlist, if L1 is not listed as the first list, place the cursor on the name of the list, press 2nd STAT and then 1 to select L1.

8. On the line Ylist, if L2 is not listed as the first list, place the cursor on the name of the list, press 2nd STAT and then 2 to select L2.

9. Press ZOOM and then 7 to select ZoomStat.

10. Press GRAPH to see the scatterplot.

11. Press 2nd STAT and then use the right arrow to highlight CALC.

12. Press 6 to select Quadratic Regression. (Other TI calculators will have more choices than the ones given.) Press ENTER. The best quadratic formula will be given.

13. Press Y= and clear any functions that may be there. Put the cursor after Y1= and press 2nd VARS. Then press 3 to select Statistics. Use use the right arrow to highlight EQ. Press 1 to select RegEQ.

14. Press GRAPH to graph the scatterplot together with the curve given by the regression function.

Other Websites and Software

1. `http://www.ms.uky.edu/~lee/outreach/outreach.html` — Mathematics Education and Outreach Resources.

2. `http://www.geogebra.org/cms` — GeoGebra. Free dynamic geometry software for algebraic and geometric plotting and sketching.

3. `http://math.exeter.edu/rparris/default.html` — Peanut Software. A nice collection of free software.

   (a) Wingeom — Dynamic geometry software

   (b) Winarc — Various games of mathematical interest

   (c) Winmat — Working with matrices

   (d) Winplot — Two and three dimensional graphing

   (e) Winfeed — Fractals

   (f) Wincalc — Working with numbers of arbitrary length

   (g) Winstats — Probability and statistics

   (h) Windisc — Discrete math

   (i) Winwordy — Word puzzles

The TI Probability Simulator

A set of activities for use with the TI Probability Simulator application can be downloaded from this website:

```
http://education.ti.com/educationportal/sites/US/nonProductSingle/
activitybook_probsimapp_exploring.html
```

1. Law of Large Numbers: Equal Opportunities

2. Law of Large Numbers: A Weighty Decision

3. Law of Large Numbers: Adding it Up

4. To Replace or Not to Replace: That is the Question

5. The Spread of Disease

6. Off to the Races

7. Gambler's Fallacy: Longest Streaks

8. Gambler's Fallacy: Lucky Streaks and Slumps

OTHER RESOURCES