History of Mathematics #10

1. Before 11 pm, Sunday, March 26. Go to the Forum "Series" and make at least one substantive contribution by 11 pm, Sunday, March 26, and at least one substantive response to others' postings before class on Tuesday, March 28. Write about the following:

   Reflect on some of the important historical developments revolving around series, and to what extent the nature of these developments could or should be mirrored in K-16 curricula.

2. Before Tuesday, March 28.

   (a) Read Dunham Chapter 9. Skim Boyer Chapter 21.

   (b) Think about the following questions for discussion at the Centra session:

      i. What is the earliest informal encounter with series in the K–16 curriculum? Does this correspond to historical development?

      ii. Be prepared to describe and discuss the following items. When are they encountered in K–16 curriculum? When do they make their appearances in history?

         A. Zeno's paradox
         B. Repeating decimals
         C. Non-repeating decimals
         D. Arithmetic series
         E. Geometric series
         F. Power series
         G. Harmonic series
         H. Taylor series
         I. Generating functions
         J. Fourier series

3. Tuesday, March 28, 7–9 pm. Attend the Centra session to discuss the readings, forum, and comments and questions on the assigned homework due on Saturday.

4. Homework problems due Saturday, April 1, 11 pm, uploaded on the moodle site (preferred method) or submitted to the email address mathhist@ms.uky.edu.

(a) Look up Buffon's needle problem, and make a precise statement of the result (it involves π in probability in perhaps a surprising way).

(b) Do not use outside sources for this problem, though you may talk with each other. Derive the Taylor series for $\sin x$ given in Dunham:

$$\sin x = \frac{x^1}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots .$$

(Recall that the Taylor series expansion of a function $f(x)$ about the point $x = 0$ is given by

$$f(x) = \frac{f(0)x^0}{0!} + \frac{f'(0)x^1}{1!} + \frac{f''(0)x^2}{2!} + \frac{f^{(3)}(0)x^3}{3!} + \cdots + \frac{f^{(k)}(0)x^k}{k!} + \cdots,$$

that the derivative of $\sin x$ is $\cos x$, and that the derivative of $\cos x$ is $-\sin x$.)

(c) Study page 446 of Boyer in which the divergence of the Harmonic series is used to prove the infinitude of primes. Study and then rewrite this explanation in your own words, filling in any necessary details.

(d) Do not use outside sources for this problem, though you may talk with each other. Consider the sequence $1, 1, 5, 13, 41, \ldots$ defined by $a_0 = 1$, $a_1 = 1$, and $a_n = 3a_{n-2} + 2a_{n-1}$ for all $n \geq 2$. In this problem we will use series to derive a formula for a_n.

i. Define the series

$$\begin{aligned} y &= \sum_{n=0}^{\infty} a_n x^n \\ &= 1 + x + 5x^2 + 13x^3 + 41x^4 + \cdots . \end{aligned}$$

Operating naively on this series term by term, show that $y - (2x)y - (3x^2)y = 1 - x$. Conclude that

$$y = \frac{1 - x}{-3x^2 - 2x + 1}.$$

ii. Find real numbers A and B so that

$$\frac{1 - x}{-3x^2 - 2x + 1} = \frac{A}{1 - 3x} + \frac{B}{1 + x}.$$

iii. Use standard facts about geometric series to prove

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots$$

and

$$\frac{1}{1-3x} = 1 + 3x + (3x)^2 + (3x)^3 + \cdots.$$

iv. Put everything together, using (4(d)ii) and (4(d)iii), comparing the coefficients of the powers of x term by term, to conclude

$$a_n = \frac{1}{2}(3^n + (-1)^n).$$

(e) Extra Credit. Follow a process similar to that of the previous problem to derive a formula for the nth Fibonacci number.