Some Notes on Homework #9

1. The cycloid problem. Here is a way to derive the equation of the cycloid that is equivalent to what all of you did, but perhaps a bit shorter (including no need for multiple cases):

First, assume the circle is centered at (0, 0) and calculate the coordinates of the point \((x, y)\) resulting from the clockwise (negative) rotation of the point \((0, -a)\) (hence initially at an angle of \(-\pi/2\)) through the angle \(z\):

\[
(x, y) = (a \cos(-\pi/2 - z), a \sin(-\pi/2 - z)) = (-a \sin z, -a \cos z)
\]

Now translate the circle up by \(a\) units and to the right by \(az\) units to put it in its final position:

\[
(x, y) = (-a \sin z, -a \cos z) + (az, a) = (az - a \sin z, a - a \cos z).
\]

2. The tautochrone.

One minor point in physics: Because rolling objects have kinetic energy associated with both the rolling (rotational kinetic energy) as well as the velocity along the curve, I am not sure if the tautochrone (or the brachistochrone) property works for rolling objects (but I admit that I have not done the calculations). Ideally we must consider frictionless sliding objects.

3. The bricks.

If you poke around on the web, you can find a photo of bricks with large overhang. For example, I googled “harmonic brick photo” and immediately found


Amazing what you can find on the web!

4. The non-transitive dice.

Here is how I remember the numbering for the dice. Begin with the standard \(3 \times 3\) magic square:

\[
\begin{array}{|c|c|c|}
\hline
8 & 1 & 6 \\
\hline
3 & 5 & 7 \\
\hline
4 & 9 & 2 \\
\hline
\end{array}
\]
You can use the numbers in the first row for die A, just using every number twice: 1, 1, 6, 6, 8, 8. Similarly for dice B and C. But to avoid repetition of numbers on the dice, I doubled the numbers, and subtracted 1 from one of the repetitions: 1, 2, 11, 12, 15, 16 for die A, etc.