MA 111 Review for Exam 5

Exam 5 (given in class on Monday, November 16) will cover Unit 5: The Mathematics of Money. Use Chapter 10 in the textbook and the class slides as study tools.

Can you work each homework problem correctly and quickly, providing explanations and justifications, without looking at the notes?

I will provide a formula sheet. You will need to know which formula to use for each problem. You should be familiar with the following key ideas:

- Understand simple interest. Be able to solve for any of the variables in the simple interest formula if given the other three. Here is the simple interest formula:
  
  \[ F = P(1 + rt). \]

- Understand the concept of compound interest. Be able to use the compound interest formula to solve for the variable \( P \) or \( F \). Here is the compound interest formula:
  
  \[ F = P \left(1 + \frac{r}{n}\right)^{nt}. \]

- Know the definition of APY and be able to find the APY for a given investment. Be able to use APY to compare investments. To find the APY, just check the percent growth of $1 in one year.

- Understand what a geometric sequence is, and know the definition of the initial term and the common ratio.

- Be able to find a term in a geometric sequence, given the initial term \( P \) and the common ratio \( c \).

- Know how to use the geometric sum formula:
  
  \[ P + cP + c^2P + \cdots + c^{N-1}P = P \left[\frac{c^N - 1}{c - 1}\right], \]

  including for problems like
  
  \[ 2(1.03) + 2(1.03)^2 + \cdots + 2(1.03)^{100} \]

  and
  
  \[ 2/(1.03) + 2/(1.03)^2 + \cdots + 2/(1.03)^{100} \]

- Understand what an annuity is, and be able to distinguish between a deferred annuity and an installment loan.

- Know how to use the deferred annuity formula:

  \[ F = L \left[\frac{(1+p)^T - 1}{p}\right]. \]

  Use \( L = P(1 + p) \) if the last payment is at the beginning of the last pay period (which is usually the case), but \( L = P \) if the last payment is at the end of the last pay period. Be able to use the formula to solve for \( F \) or \( P \).
Know how to use the amortization formula:

\[ P = F q \left[ \frac{q^T - 1}{q - 1} \right], \]

where \( q = \frac{1}{1+p} \). It is usually the case that the first payment is at the end of the first pay period. Be able to solve for \( F \) or \( P \). If it happens that the first payment is at the beginning of the first pay period, use the formula

\[ P = F \left[ \frac{q^T - 1}{q - 1} \right]. \]

Be able to compare different financing options in the context of buying, say, a car or house.
Practice Problems

Also see the odd exercises on pages 393–398. Check the back of the book for the correct answers.

1. Your parents purchased a government bond 18 years ago at \(5\frac{1}{2}\%\) annual simple interest. Now it is worth $6965. How much did your parents pay for the bond?

2. Suppose you deposit $250 into a savings account with a 5\%\ APR.
   (a) If the interest is compounded yearly, how much money will be in the account in 20 years?
   (b) If the interest is compounded daily, how much money will be in the account in 20 years?

3. Use the APY to answer the following: Which of the following investments is better: a 4\%\ APR compounded monthly, or a 4.25\%\ APR compounded annually?

4. A savings account at your bank offers a 4\%\ APR compounded weekly.
   (a) If you make only one deposit of $1500, and then don’t touch the account, how much money will be in the account in 3 years?
   (b) If instead you deposit $15 each week, how much money will be in the account in 3 years? (Assume that you deposit money at the beginning of the week, and interest is added at the end of the week.)

5. Consider the geometric sequence with first two term \(G_0 = 2\) and \(G_1 = 5\).
   (a) Find the common ratio \(c\).
   (b) Give an explicit formula for \(G_N\).
   (c) Find \(G_7\).
   (d) Compute \(G_0 + G_1 + G_2 + \cdots + G_{29} + G_{30}\).

6. You would like to save $7,000 for an overseas trip that will begin in 18 months. If you plan to make a daily deposit into a savings account with an APR of 7\%, compounded daily, how much should your daily deposit be in order to meet your goal? Assume there are exactly 30 days in each month, and that the last deposit generates no interest.

7. (a) Suppose you wish to purchase a car that costs $11,500. If you can get a 10 year loan with a 5\%\ APR compounded monthly, how much will your monthly payment be?
    (b) With such monthly payments, how much interest will you pay over the 10 years?

8. Your bank approves you for a 30 year home loan with a 6\%\ APR compounded monthly and monthly payments of $800. What price can you afford for your new house?