

## MA111 – Homework #1 Short Solutions

### Chapter 1

2. (a)

Number of voters	4	5	6	2
1st choice	A	B	C	A
2nd choice	D	C	A	C
3rd choice	B	D	D	D
4th choice	C	A	B	B

(b) None; A and C each has six votes.

(c) D

4. (a) 1240

(b) 621. Half of 1240 is 620, and a majority candidate must have more votes than this number.

(c) B

6. (a) A and C

(b)

Number of voters	745	495
1st choice	B	D
2nd choice	D	B

(c) B

8. Note that remaining voters,  $225 + 675 = 900$ , constitute 40% of the voters. So 20% of the voters equals 450.

Number of voters	450	900	225	675
1st choice	A	B	C	C
2nd choice	C	C	B	A
3rd choice	B	A	A	B

12. (a) A and B tie.

(b) B

(c) A

14. (a) 28 votes guarantee B is the outright winner, because then B would have 46 votes, and the most votes that C could get would be 42 plus 2 more, or 44.

(b) 20 votes guarantee D is the outright winner, because then D would have 54 votes, and the most votes that C could get would be 42 plus 10 more, or 52.

16. (a) 257. What this means is that it is *possible* (though not *guaranteed*) to win the plurality with 257 votes—the other candidates could each have 256 votes. But it is *impossible* to win the plurality with 256 votes—someone else would have more than 256 votes. The number 257 is gotten by dividing the total number of voters by the total number of candidates:  $1025/4 = 256.25$ . To be *possible* to win with a plurality, a candidate would need more votes than this number—at least 257.

- (b) 8. By the previous reasoning, if there are  $N$  candidates, we need  $1025/N \approx 129$ , or  $N \approx 1025/129 \approx 7.9$ . And indeed if  $N = 8$ , then to be *possible* to win with a plurality, a candidate would need more than  $1025/8 = 128.125$  votes—at least 129.
- (c) 5. By the previous reasoning, if there are  $N$  candidates, we need  $1025/N \approx 206$ , or  $N \approx 1025/206 \approx 4.98$ . And indeed if  $N = 5$ , then to be *possible* to win with a plurality, a candidate would need *more than*  $1025/5 = 205$  votes—at least 206.
20. (a) D  
(b) C has a majority of the first-place votes but does not win the election.  
(c) C, having a majority of the first-place votes, is a Condorcet candidate but does not win the election.
24. (a) 100  
(b) 20  
(c) 15  
(d) 300  
(e) 49
58. (a) 24. There are 4 choices for the first choice, times 3 choices for the second choice (once the first choice is made), times 2 choices for the third choice (once the first and second choices are made), times 1 choice for the fourth choice (once the first, second, and third choices are made).  
(b) 12. There are 4 choices for the first choice, times 3 choices for the second choice.