## MA 111 Worksheet on Divider-Chooser

(Created by Paul Koester.)

1. Matt, Nat, Pat, and Kat are to divide a cake. They decide to use the lone-divider method to divide the cake. Matt is chosen to be the divider, and he divides the cake into four slices. The table shows how each person values each of the four slices.

	А	В	С	D
Matt	25%	25%	25%	25%
Nat	21%	31%	20%	28%
Pat	45%	10%	15%	30%
Kat	20%	25%	20%	35%

(a) Which of the four slices will Nat include in his bidset?

- (b) Which of the four slices will Pat include in her bidset?
- (c) Which of the four slices will Kat include in her bidset?
- (d) Find a fair allocation of the four slices.
- (e) Find another fair allocation of the four slices.
- (f) How many ways can these four slices be fairly allocated to Matt, Nat, Pat, and Kat?

2. Clara, Farrah, Kara, Sarah, and Tara have inherited a large plot of land that is to be divided equally amongst themselves. They each value different parts of the land differently (for example, Clara grew up in the Sahara and wants the hot, sandy part of the land whereas Farrah likes to hike and wants the heavily wooded part of the land). They decide to use the lone-divider method to break up the land into fair shares. Suppose Clara is the divider, and she divides the land into pieces A, B, C, D, and E. Suppose the bidsets are

Farrah:  $\{A, C\}$ Kara:  $\{A, E\}$ Sarah:  $\{A, B, C, E\}$ Tara:  $\{C, E\}$ 

- (a) Which piece must Clara receive?
- (b) Which piece must Sarah receive?
- (c) Find a fair-division for the remaining players.
- (d) Find another fair-division for the remaining players.

(All those years of playing Sudoku are finally paying off!)

**3.** Ed, Ned, Jed, and Ted recently received an inheritance after their rich Uncle Zed ingested lead, and is now dead. Oh, what dread! Zed's will stipulates that his estate be "divided equally amongst my four nephews." Each of the nephews values the various items in his estate differently, so the nephews decide to use the lone-divider method to divvy up the estate. One of the nephews divided the estate into four pieces. The table gives partial information as to how each person values each of the four slices. Furthermore, you know that Ted likes A and D equally, he likes B and C equally, and he likes B four times as much as he likes A.

	А	В	С	D
Ed	20%	30%		20%
Ned		25%		
Jed		30%	40%	10%
Ted				

- (a) Fill in the remaining values in the table.
- (b) Which of the four nephews was the divider?
- (c) Find the bidset for each of the three choosers.

(d) Can A, B, C, D be distributed fairly between the four nephews? If so, find such a distribution. If not, explain why.