

# MA 111 Notes on Lone-Divider: The Case of a Conflict

## Some Answers

December 2, 2011

Jon, Ron, and Don are going to split a sub sandwich. The sandwich is 9 inches long. The first three inches is roast beef, the second three inches is turkey, and the last three inches is vegetarian. You are given

- Jon is indifferent. He likes everything equally.
- Ron likes turkey and vegetarian equally, and he likes roast beef twice as much as he likes turkey.
- Don likes roast beef three times more than he likes turkey and he doesn't like vegetarian at all.

They will divide the sandwich using the lone divider method.

Suppose Jon is selected to play the role of the divider.

- (a) How will Jon divide the sandwich?

One approach that some find helpful in this kind of problem is to assign an artificial price to the object being divided. It doesn't matter that the price is not realistic—we are just going to compare relative values. And it is nice to use a number with lots of divisors, like \$360 (definitely not realistic!)

John values each section equally, so to him, each is worth  $\$360/3 = \$120$ . So he will divide the sandwich into 3 inches of roast beef, three inches of turkey, and 3 inches of vegetarian, each worth \$120.

- (b) How does Ron value the three slices, and which slices will Ron include in his bidset?

Based on Ron's likes, he values turkey and vegetarian equally, say each is valued  $x$ , and roast beef twice as much as turkey, so  $2x$ . These must sum to \$360, so  $2x + x + x = 360$ , which means  $4x = 360$  and so  $x = 90$ . Thus Ron values roast beef as \$180 and turkey and vegetarian each as \$90. Now Ron wants one third of the total value of the sandwich, so is entitled to at least \$120 worth. Therefore, he is only going to place the roast beef slice in his bidset.

- (c) How does Don value the three slices, and which slices will Don include in his bidset?

Based on Don's likes, he values vegetarian as \$0. Suppose he values turkey as  $x$ . Then he values roast beef as  $3x$ . Again these must sum to \$360, so  $3x + x = 360$ , which means  $4x = 360$  and so  $x = 90$ . Thus Don values roast beef as \$270 and turkey as \$90. Now Don wants one third of the total value of the sandwich, so is entitled to at least \$120 worth. Therefore, he is only going to place the roast beef in his bidset.

There is a conflict! Ron and Don each are fighting for the roast beef slice. How do we resolve this conflict? Jon will be given one of the unwanted slices (the so-called U pieces). Let's assume Jon receives the vegetarian slice. Ron and Don will now enter into a fair division game to divide the 6 inch, roast beef and turkey sub. Now, all fair division methods reduce to the two person "I cut-you choose" method. Suppose Don is selected to be the divider. (At this point, the vegetarian slice is gone; Don and Ron are each trying to get at least 50% of the six inch sub. At the end of the problem, we will see how much their slices are worth in terms of the original sub.)

- (d) How will Don divide the sandwich?

Don is faced with dividing a 6 inch sandwich which is 3 inches of roast beef (worth \$270 to him) and 3 inches of turkey (worth \$90 to him). This sandwich is worth \$360 to him. He must divide this into two shares of equal value to him. Since he values roast beef more than turkey, you can see that one share will consist of a certain amount of roast beef, say  $x$  inches, while the other share will consist of the rest of the roast beef and all of the turkey.

The value of the first share to Don is  $\frac{x}{3}$  (the fraction of the roast beef) times \$270 (the value of the roast beef), hence  $\frac{x}{3} \cdot 270 = 90x$ . This is supposed to equal half of \$360 (the value of the sandwich), so  $90x = 180$ , which means  $x = 2$  inches. So he is going to create

two shares: The first share will be 2 inches of roast beef (which we just saw is worth \$180), and the second share is 1 inch of roast beef and 3 inches of turkey. So the second share is worth  $\frac{1}{3} \cdot 270$  (the fraction of the roast beef times the value of the roast beef) plus  $1 \cdot 90$  (the fraction of the turkey times the value of the turkey). Together these sum to \$180. So both shares are worth \$180 to Don.

- (e) How does Ron value each of the two slices, and which slice will Ron choose?

To Ron, the 6 inch sandwich is worth \$180 of roast beef plus \$90 of turkey, or a total of \$270. The first share to him is worth  $\frac{2}{3} \cdot 180$  (the fraction of the roast beef times the value of the roast beef), which equals \$120. The second share to him is worth  $\frac{1}{3} \cdot 180$  (the fraction of the roast beef times the value of the roast beef) plus  $1 \cdot 90$  (the fraction of the turkey times the value of the turkey), which together equals \$150. He feels that he is entitled to a share worth at least half of the \$270, so he will choose the second share, worth \$150 in his value system.

Jon, Ron, and Don now each has a slice of the original 9 inch sub. Did they each get at least one third of the value of the whole sub?

- (f) Jon received the vegetarian slice, which Jon thinks is  $\frac{1}{3}$  of the value of the whole sub.

So Jon's share is worth \$120 to him, which is  $\frac{1}{3}$  of the \$360 value of the original sandwich.

- (g) How much is Don's slice worth (as a percentage or fraction of the 9 inch sub)?

As we have seen, Don values his final share as \$180, which as a fraction of the value of the original sandwich is the fraction  $\frac{180}{360} = \frac{1}{2}$  of the original sandwich.

- (h) How much is Ron's slice worth (as a percentage or fraction of the 9 inch sub)?

As we have seen, Ron values his final share as \$150, which as a fraction of the value of the original sandwich is the fraction  $\frac{150}{360} = \frac{5}{12}$  of the original sandwich.

So each person feels that they have received at least  $\frac{1}{3}$  of the value of the original sandwich.