

MA 111 Review for Exam 1

Exam 1 (given in class on Tuesday, February 9) will cover Using Numbers and Quantities, and Percent and Percent Change, discussed in Sections 1 and 2 of the text.

Can you work each homework, worksheet, and quiz problem *correctly* and *quickly*, providing explanations and justifications, without looking at the text or your notes?

Have you carefully studied the material in the text?

You should be familiar with the following key ideas:

- Understand the difference between number and quantity.
- Know how to convert units (e.g., miles/hour to feet/sec).
- Know how to compare quantities with ratios and with percents.
- Know how to understand, or make better sense, of very large or very small quantities.
- Understand the importance of correct units.
- Understand what percent means.
- Know what the base of a percent is, and be able to identify the base of a percent in context.
- Be able to find different variables in a percentage calculation (e.g., use the base and the result to find the percent).
- Understand how changes in numbers can be measured by percents (percent change).
- Know how to increase or decrease a number by a percentage, and how to reverse this process.
- Understand the difference between (1) change, (2) percent change, (3) change in percent (percentage points), and (4) percent change of percent.
- Be able to interpret a short passage and analyze the given information about numbers, quantities, percents, and percent change.

Practice Problems

1. I enjoy riding my recumbent bicycle at 12 miles per hour. What is my speed in yards per minute?

$$\frac{12 \text{ miles}}{1 \text{ hour}} \times \frac{5280 \text{ feet}}{1 \text{ mile}} \times \frac{1 \text{ yard}}{3 \text{ feet}} \times \frac{1 \text{ hour}}{60 \text{ minutes}} = \frac{352 \text{ feet}}{1 \text{ minute}}$$

2. If light can travel 186,000 miles per second, how many miles can it travel in one year? (Assume one year has 365.25 days.)

$$\frac{186,000 \text{ miles}}{1 \text{ second}} \times \frac{60 \text{ seconds}}{1 \text{ minute}} \times \frac{60 \text{ minutes}}{1 \text{ hour}} \times \frac{24 \text{ hours}}{1 \text{ day}} \times \frac{365.25 \text{ days}}{1 \text{ year}} \approx \frac{3.87 \text{ trillion miles}}{1 \text{ year}}$$

3. Compare the numbers \$23.9 billion and \$4.57 trillion as a ratio and as a percent. (Your answer should be in the form of complete sentences.)

\$4.57 trillion is about 191 times as large as \$23.9 billion, or \$4.57 trillion is about 19100% of \$23.9 billion. \$23.9 billion is about $\frac{1}{191}$ of \$4.57 trillion, or \$23.9 billion is about 0.52% of \$4.57 trillion.

4. Find an effective way to make sense of the size of Obama's proposed \$3.8 trillion federal budget.

Examples: Using the estimate that the world's population is about 6.67 billion, with \$3.8 trillion you could give each person on earth about \$570. Using the estimate that the U.S. population is about 312 million, each person's share of the debt in this country is about \$12,000.

5. Find an effective way to make sense of the size of a water molecule, 0.2 nanometers, which equals 0.000000002 meters.

Example: Using the estimate of a human hair being 0.00002 meters wide, it would take about 100,000 water molecules to achieve the width of one hair.

6. Express each of the following percentages as a decimal.

(a) 43%

0.43

(b) 1.5%

.015

(c) 250%

2.5

(d) 0.13%

.0013

7. If your score on a test was 62 points out of 70, express the score as a percentage.

$$\frac{62}{70} = \frac{P}{100}, \text{ so } P = \frac{62}{70} \times 100 \approx 89\%.$$

8. What is 37% of 56?

$$\frac{A}{56} = \frac{37}{100}, \text{ so } A = \frac{37}{100} \times 56 = 20.72.$$

9. 78 is what percent of 52?

$$\frac{78}{52} = \frac{P}{100}, \text{ so } P = \frac{78}{52} \times 100 = 150\%.$$

10. 123 is 17% of what?

$$\frac{123}{N} = \frac{17}{100}, \text{ so } N = \frac{123 \times 100}{17} \approx 723.5.$$

11. Out of a class of 35 students, 28 students passed the midterm exam. What percentage of the class failed the midterm?

If 28 students passed, then 7 students failed. $\frac{7}{35} = \frac{P}{100}$, so $P = \frac{7}{35} \times 100 = 20\%$.

12. If your income in one year is \$65000 and you pay \$10000 in income tax, what is the tax rate?

$$\frac{10000}{65000} = \frac{P}{100}, \text{ so } P = \frac{10000}{65000} \times 100 \approx 15.4\%.$$

13. If you buy a book that costs \$19.95, and the sales tax is 6%, what amount do you owe?

$$\frac{A}{\$19.95} = \frac{6}{100}, \text{ so } A = \frac{6}{100} \times \$19.95 \approx \$1.20.$$

14. If you read that the income tax is \$230 per \$1000 in income, what is the tax rate as a percent?

$$\frac{230}{1000} = \frac{P}{100}, \text{ so } P = \frac{230}{1000} \times 100 = 23\%.$$

15. Refer to the previous problem. At that tax rate, if you owed \$34500 in income tax, how much was your income?

$$\frac{\$34500}{N} = \frac{23}{100}, \text{ so } N = \frac{\$34500 \times 100}{23} = \$150,000.$$

16. In 2000, tuition at UK was \$6550. In 2001, tuition increased by 7%. How much was tuition at UK in 2001?

Use $B = A(1 + \frac{P}{100})$. Here, $B = \$6550(1 + \frac{7}{100}) = \7008.50 .

17. A computer that usually sells for \$830 is on sale for 15% off. How much does the computer cost?

Use $B = A(1 - \frac{P}{100})$. Here, $B = \$830(1 - \frac{15}{100}) = \705.50 .

18. One year ago, a gallon of gas sold for \$2.82. Now the price of a gallon of gas is \$2.63. By what percent has the price decreased over the past year?

Use $P = \frac{B-A}{A} \times 100$. Here, $P = \frac{\$2.63 - \$2.82}{\$2.82} \times 100 \approx -6.74\%$. So the price has decreased by about 6.74%.

19. The population of Pleasantville this year is 5,900. This is a 3% increase over last year's population. What was the size of the population of Pleasantville last year?

Use $B = A(1 + \frac{P}{100})$. Here, $5900 = A(1 + \frac{3}{100})$, so $A = \frac{5900}{1.03} = 5728$.

20. (a) If 300 is decreased by 5%, what is the result?

Use $B = A(1 - \frac{P}{100})$. Here, $B = 300(1 - \frac{5}{100}) = 285$.

- (b) If 300 is decreased by 15% and then the result is increased by 10%, what is the final result?

First use $B = A(1 - \frac{15}{100})$. Here, $B = 300(1 - \frac{15}{100}) = 255$. So if 300 is decreased by 15%, the result is 255. Now increase 255 by 10%, using $B = A(1 + \frac{P}{100})$. Here, $B = 255(1 + \frac{10}{100}) = 280.5$.

Alternatively, you can combine the two operations, and calculate $300(1 - \frac{15}{100})(1 + \frac{10}{100}) = 280.5$.

- (c) If a number is decreased by 15% and then the result is increased by 10%, resulting in a final value of 300, what was the initial number?

Longer way: First reverse the increase of 10% using $B = A(1 + \frac{P}{100})$. Here, $300 = A(1 + \frac{10}{100})$, so $A = \frac{300}{1.1} \approx 272.7$. Next, reverse the decrease of 15% using $B = A(1 - \frac{P}{100})$. Here, $272.7 = A(1 - \frac{15}{100})$, so $A = \frac{272.7}{0.85} \approx 320.8$.

Shorter way: Combining the two operations, starting with the initial unknown amount A , $A(1 - \frac{15}{100})(1 + \frac{10}{100}) = 300$. So $A = \frac{300}{0.85 \times 1.1} \approx 320.8$.

21. In 2008 the freshman class had 3000 students and 42% of the freshman class were enrolled in a math class. In 2009 the freshman class had 3500 students and 51% of the freshman class were enrolled in a math class.

- (a) What is change in the number of students in the freshman class who were enrolled in a math class?

In 2008 the number of students enrolled in a math class was 42% of 3000, which equals $\frac{42}{100} \times 3000 = 1260$. In 2009 the number of students enrolled in a math class was 51% of 3500, which equals $\frac{51}{100} \times 3500 = 1785$. So the change in the number of students enrolled in a math class is $1785 - 1260 = 525$.

- (b) What is the percent change in the number of students in the freshman class who were enrolled in a math class?

Use $P = \frac{B-A}{A} \times 100$. Here, $P = \frac{1785-1260}{1260} \times 100 \approx 41.7\%$.

- (c) What is the change in the percent of students in the freshman class who were enrolled in a math class?

The change in the percent is $51\% - 42\% = 9\%$, or 9 percentage points.

- (d) What is the percent change in the percent of students in the freshman class who were enrolled in a math class?

Use $P = \frac{B-A}{A} \times 100$. Here, $P = \frac{51-42}{42} \times 100 \approx 21.4\%$.

22. The town council decides to reduce the sales tax from 6% to 5%. By what percent did the sales tax decrease?

The percent change in the percent is $\frac{5-6}{6} \times 100 \approx -16.7\%$. So the sales tax decreased by 16.7% (even though it only decreased by 1 percentage point).

23. In a poll taken in September, Governor Jane Citizen was favored by 49% of the respondents. In a poll taken in October, Gov. Citizen was favored by 55% of the respondents. How many percentage points did Gov. Citizen gain from September to October? By what percent did the favorable responses increase?

Gov. Citizen gained 6 percentage points. But Gov. Citizen's favorable responses (assuming the same number respondents, which probably is not likely, so this answer is likely to be incorrect in reality) increased by $\frac{55-49}{49} \times 100 \approx 12.2\%$.

24. Read this excerpt from <http://threesixty360.wordpress.com/2008/11/12/math-confusion-in-the-news-percent>:

Last week Governor Arnold Schwarzenegger proposed a temporary (3-year) sales tax increase in California to help close the budget deficit. Some newspapers, however, are mixing up the amount of the increase in an effort to get the news out.

Error #1: “Governor Proposes 1.5 Percent Sales Tax Hike” from MyFox Los Angeles

The proposed increase isn’t actually 1.5 percent (which wouldn’t be all that much). It’s 1.5 percentage points, which makes it about a 20% increase from the current 7.25% state sales tax. I suspect that most people understand what the headline intends, however, because using “percent” instead of “percentage point” is fairly common. (Kudos to the LA Times for being precise in their story!)

- (a) Explain what calculation is used to arrive at the “about 20% increase” statement and give a more precise number rounded to one decimal place.

The calculation used was to calculate the percent increase in the percent; namely, $\frac{8.75-7.25}{7.25} \times 100 \approx 20.7\%$.

- (b) What would the new tax rate be if the tax increase were truly a 1.5% increase, rather than increase of 1.5 percentage points? *Round your answer to 2 decimal places.*

If you increase 7.25 by 1.5%, the result would be $7.25(1 + \frac{1.5}{100}) \approx 7.36$. So the new tax rate would be 7.36%.