Chapter 6
Notes for Instructors

Content

This focus of this chapter is the rational number system. Although the chapter contains only three sections, you should allot a full week for section 6.2. Students tend to have more difficulty with fractions than they do with any other topic covered in this class.

Manipulatives

We have several types of manipulatives that can be used to model fractions including Frax Pax ™Squares, Cuisenaire® rods, pattern blocks, and colored counters (the circular pieces that are yellow on one side and red on the other).

- It is possible to use some of the Frax Pax ™to model multiplication with the area model. This model will be discussed momentarily. If you do this, you may need to specify that your unit is not the large square. If you want to use the large square as the unit, you could combine several boxes of Frax Pax ™if one of your factors is greater than one.
- The Cuisenaire® rods can be used to do an activity similar to the Cooperative Investigation shown on page 348 of the textbook. Questions 2 and 3 of this investigation were both challenging and enlightening for the students.
- The pattern blocks can be used to construct colored region models for fractions like those shown on page 347 and in problems 8, 10, and 25 of Section 6.1. You could also write questions similar to those in the Cooperative Investigation on page 348 that involve pattern blocks instead of fractions strips.
- The colored counters can be used to construct set models of fractions.

Notes and Suggestions:

Notes on Section 6.1: The Basic Concepts of Fractions and Rational Numbers

- It is imperative that students know the three things which must be well defined in order to interpret the meaning of any fraction $\frac{a}{b}$. (See the bulleted items on page 346.)
- Some of the students in this course have a lot of difficulty reading and understanding definitions. To encourage them to read definitions carefully, I like to ask, "Is $\frac{7}{6}$ in simplest form?" Most of my students thought that it needed to be converted to a mixed number, but this is not so. Because the $\text{GCD}(7, 6) = 1$ and 6 is positive, the fraction $\frac{7}{6}$ is in simplest form. (See the definition on page 352.)
- I think it is important to emphasize that you can find a common denominator without necessarily finding a least common denominator for a set of fractions. Moreover, any

common denominator will often be sufficient for solving a certain problem. For example, if you need to add two fractions, it is sufficient to find any common denominator for the two fractions. Finding the least common denominator is, in many cases, more of a stylistic issue than anything else.

- I think the Order Relation on the Rational Numbers is best understood by finding a common denominator for the pair of fractions $\frac{a}{b}$ and $\frac{c}{d}$. I found that it was a worthwhile exercise to have students explain the Order Relation on the Rational Numbers by using common denominators. Without this explanation, the inequality $ad < bc$ has little meaning to students.

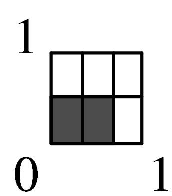
Notes on Section 6.2: The Arithmetic of Rational Numbers

- Students should be able to demonstrate why $\frac{a}{b} + \frac{c}{d} \neq \frac{a+c}{b+d}$ for given values of a, b, c, and d. For example, we can use the Frax Pax Squares to *see* that $\frac{1}{2} + \frac{1}{3} \neq \frac{2}{5}$ since the area covered by the $\frac{1}{2}$ rectangle and the $\frac{1}{3}$ rectangle together is much larger than the area covered by two of the $\frac{1}{5}$ rectangles.

- Since common denominators are required for the addition of fractions and subtraction is defined according to the missing addend model, it follows that common denominators are required for the subtraction of fractions.

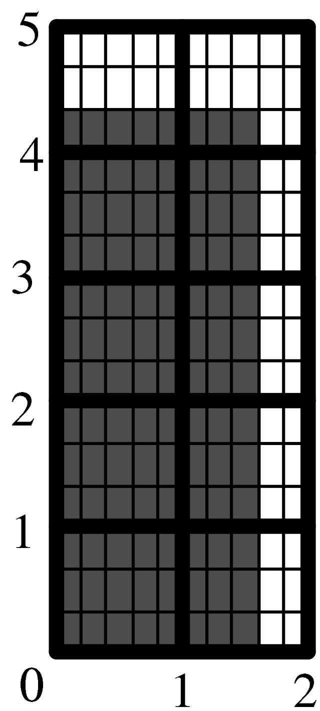
- I really like the way that the area model can be used to multiply fractions. To understand this model, students need to understand that the unit is the 1×1 square because it has an area of 1 square unit. When you have established this, the other information needed to interpret the fractional answer (see the bulleted items on page 346) follows easily. Two examples are given below.

  - **Example 1:** $\frac{1}{2} \times \frac{2}{3}$
    To compute $\frac{1}{2} \times \frac{2}{3}$ using the area model, you need a rectangle of dimensions $\frac{1}{2}$ by $\frac{2}{3}$. To obtain such a rectangle, you must divide the unit square into six equal pieces as shown below. Two these small pieces compose the $\frac{1}{2}$ by $\frac{2}{3}$ rectangle. Hence the area of the $\frac{1}{2}$ by $\frac{2}{3}$ rectangle is $\frac{2}{6}$ square units and $\frac{1}{2} \times \frac{2}{3} = \frac{2}{6}$.



  - **Example 2:** $4\frac{1}{3} \times 1\frac{3}{5}$
    To compute $4\frac{1}{3} \times 1\frac{3}{5}$ using the area model, you need a rectangle of dimensions $4\frac{1}{3}$ by $1\frac{3}{5}$. To obtain such a rectangle, you must divide each unit square into fifteen equal pieces as shown below. Note that the diagram below contains ten units.

The $4\frac{1}{3}$ by $1\frac{3}{5}$ rectangle is composed of four unit squares and forty-four of the small rectangles. Since fifteen of the small rectangles can be used to construct a unit, we actually have six units and fourteen of the small rectangles. Thus the area of the $4\frac{1}{3}$ by $1\frac{3}{5}$ is $6\frac{14}{15}$ square units, and $4\frac{1}{3} \times 1\frac{3}{5} = 6\frac{14}{15}$.



Questions 11–13 in Section 6.2 test students' understanding of the area model for multiplication.

- Students should be able to explain the Invert and Multiply Algorithm for Division of Fractions by using the missing factor model for division.

Notes on Section 6.3: The Rational Number System

- Most of the properties in this section should flow easily from the corresponding properties for integers. Of course, we now have multiplicative inverses for nonzero rational numbers and closure under division of nonzero rational numbers.
- The only really new idea in this section is the Density Property of Rational Numbers, so I think this section can be covered quickly. Still I think it is important to have students identify the properties they use. Questions similar to Problems 9 and 11 in Section 6.3 test this ability.

Worksheets

At this point in the course, I moved my class to a workshop oriented class. I ran the class very much like I would have run a Math Excel workshop. I did this because I think the students learn the material better when they actually solve the problems themselves. I did not want my students to simply mimic me. I wanted them to understand the problems and their solutions. Each student became an "expert" for a small subset of the problems on a worksheet. I partitioned the worksheet problems into small subsets of problems and randomly assigned the problems to the students. (For example, the students might draw a subset from a hat.) Each student was required to write clear correct solutions for his or her subset. These solutions were placed in a folder in the Math Library for all the students to see. Students were also required to do the other problems on the worksheet, but I did not check these for correctness since they had access to the solutions in the Math Library. A student's homework score was based on correctness for his or her subset of problems and completion for the other problems. I have included three worksheets with this documentation.