Chapter 12
Notes for Instructors

Content

Measurement is the focus of this chapter. I was surprised to discover that many students had difficulty with this chapter. In retrospect, I think I really should have emphasized The Measurement Process highlighted in the green box in Section 12.1. Students should understand area as a "covering" problem and volume as a "filling" problem.

Manipulatives and Other Resources

I used Polydron and Power Solids (Relational Geosolids are similar to Power Solids) in this chapter, both of which can be found in the Mathskellar. The polydron were particularly helpful when calculating Surface area. Students really needed to see the 3-D polyhedron laid out as a 2-D net to calculate the surface area. The power solids were helpful in comparing volumes. There is rice in the Mathskellar that can be used to fill the Power Solids, but I think it is best to use water if at all possible. It is difficult to level the rice, making the visual comparison of the volumes of the cone, cylinder, and sphere a little less than accurate.

Tangrams are discussed in Section 12.1. After I taught this lesson, I discovered that there is a set of overhead tangrams in the manipulative kits. There should be at least one of these kits in the Mathskellar. It is in a black ETA bag.

There is a copy of the Connected Mathematics Sequence in the Mathskellar. It a junior high curriculum sequence. The books *Covering and Surrounding* and *Filling and Wrapping* are particularly relevant to this chapter. It might be good to do some exercises from this sequence in class, or at least show the curriculum to your class. It is likely that some of your students will be preparing to be middle school teachers.

Notes and Suggestions:

Notes on Section 12.1: *The Measurement Process*

- I think it is important to use non-standard units of measurement in this section to really illustrate the measurement process. The *pen* unit discussed in problem number 4 of Section 12.1 can lead to a nice discussion about tilings.

- Some of the students in my class were familiar with Unit analysis because of Chemistry class or another science class, but others had difficulty with this concept. I would say the split in my class was about 50/50. When covering Unit Analysis, I think it is important to remind students that one is the multiplicative identity. Since we are multiplying by one, we actually do not change our original quantity, only its units.

- There are blocks and multi-link cubes that could be used for problem 5 in Section 12.1.

Notes on Section 12.2: *Area and Perimeter*

- When I ask students to approximate the area of a figure on a grid, the lower bound is determined by the squares completely contained within the figure and the upper

bound is given by all the squares that intersect the figure. As the grid is refined, the lower bound and upper bound should approach each other. Note that the unit does not change as the grid is refined. For example, on question 5 of Measurement Worksheet I, students will need to use the fact that 4 small squares equals 1 unit to determine the lower and upper bounds. On question 6, they will need to use the fact that 16 small squares equals 1 unit.

- I think students should understand the area formula for a rectangle. (They may need to be reminded of the area model for multiplication.) From this, they should be able to derive the area formulas for parallelograms, triangles, and trapezoids.

- You should define π as the ratio of the Circumference of a circle to its diameter. With this definition in hand, there is some motivation for the area formula for a circle. (See the diagram at the bottom of page 773 in Section 12.2 in which sectors of a circle are rearranged to be a near-parallelogram.)

- There is another motivation for the area formula of a circle given in problem 27 of Section 12.2. I am including a diagram with a similar diagram with the documentation for this chapter.

- Students may have a bit of difficulty with the unit analysis in this section because they do not understand that $\texttt{units}^2 = \texttt{units} \times \texttt{units}$. They will need to use linear conversion factors twice to to change units of area.

- I like problems 11 and 12 in Section 12.2.

Notes on Section 12.3: *The Pythagorean Theorem*

- I am including a diagram that represents the Pythagorean Theorem for the 3-4-5 right triangle with this documentation.

- I really like problems 25 and 26 in Section 12.3. We did these exercises as a class. To do this, I magnified the shapes shown right above problem 25 in Section 12.3 on the photocopier. I magnified them several times and even used the large 11×17 paper. I had each student cut out the set of shapes. (I brought the Mathskellar scissors to class.) Then we did problems 25 and 26 together as a class.

Notes on Section 12.4: *Surface Area and Volume*

- I think it is a good idea to use nets to motivate the surface area formula for a right prism. Many of my students did not understand why perimeter came into play in this formula.

- The Power Solids (or Relational Geosolids) were helpful in understanding the volume formulas for spheres and cones.

- I think it would have been a good idea to have a deck of playing cards to illustrate the volume formula for an oblique prism. (see the discussion at the top of page 800 in Section 12.4.)
- Carl Lee has a nice Wingeom file that illustrates figure 12.26. You might ask him about it.

Worksheets

I have included three worksheets with this documentation.