

## Stretching and Shrinking Extensions

1. What precisely does it mean for two regions in the plane to be congruent?
2. What precisely does it mean for two regions in the plane to be similar?
3. Carefully draw three pentagons  $A$ ,  $B$ ,  $C$ , and  $D$  such that
  - (a)  $A$  and  $B$  are congruent,
  - (b)  $A$  and  $C$  are similar but not congruent, and
  - (c)  $A$  and  $D$  are not similar.
4.
  - (a) Suppose circle  $C_1$  has center  $(1, -2)$  and radius 3, and circle  $C_2$  has center  $(2, 7)$  and radius 5. Prove that  $C_1$  and  $C_2$  are similar by finding a function of the form  $f(x, y) = (kx + a, ky + b)$  that is a one-to-one onto function mapping  $C_1$  to  $C_2$ .
  - (b) Prove that every pair of circles is similar.
5.
  - (a) Assume that  $P_1$  is a parabola with equation  $y = 4x^2 + 4x + 2$  and  $P_2$  is a parabola with equation  $y = -9x^2 + 18x - 3$ . Prove that  $P_1$  and  $P_2$  are similar by finding a function of the form  $f(x, y) = (kx + a, ky + b)$  that is a one-to-one onto function mapping  $P_1$  to  $P_2$ .
  - (b) Prove that every pair of parabolas is similar.
6. Give good examples to illustrate the following statement: “A figure in the plane has symmetry if and only if it is congruent to itself in a nontrivial way.”
7. Consider a function of the form  $f(x, y) = (ax + by + c, dx + ey + f)$ .
  - (a) Find necessary and sufficient conditions so that  $f(X)$  is congruent to  $X$  for every region  $X$  in the plane.
  - (b) Find necessary and sufficient conditions so that  $f(X)$  is similar to  $X$  for every region  $X$  in the plane.

8. For each of the following objects, determine the formulas for the volume and surface area. Use these formulas to verify that if the object is scaled so that corresponding lengths are each multiplied by a factor of  $k$ , then the surface area is multiplied by a factor of  $k^2$  and the volume is multiplied by a factor of  $k^3$ .
  - (a) Cube of side length  $s$ .
  - (b) Rectangular box (with top and bottom) of length  $\ell$ , width  $w$ , and height  $h$ .
  - (c) Sphere of radius  $r$ .
  - (d) Cylinder (with circular top and bottom) of radius  $r$  and height  $h$ .
  - (e) Cone (with circular bottom) of radius  $r$  and height  $h$ .
  - (f) Pyramid (with square base) of base side length  $s$  and height  $h$ .
9. Find a three-dimensional reptile that is not a cube or a brick (rectangular box). Include a model, a good drawing, or an illustration with Maple, with your explanation.
10. Read the essay “On Being the Right Size.”
11. Prove that two spherical triangles on the unit sphere have corresponding congruent angles if and only if they are congruent.